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Radio Wave Propagation in Structured Ionization for Satellite and Radar Applications

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13. ABSTRACT *(Maximum 200 words)*

This report is an extension to DNA 5304D and DNA-IR-82-02 which presented the radio propagation algorithms recommended for use by DNA to calculate the properties of scintillated signals. This report covers effects related to antennas and extends the formalism to cover two component power spectra of plasma fluctuations. In addition, an improved representation of the total electron content power spectrum is included to support space radar and similar applications. Appendix E contains SUBROUTINE PROP which implements the radio propagation models.

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SUMMARY

The models and algorithms in this paper represent a synthesis of results from different works by several different individuals over the past few years. In particular, I would like to acknowledge the contributions of the following people: Dr. Walter Chesnut and of SRI International, Dr. K. C. Yeh and his colleagues at the University of Illinois at Urbana, Dr. Roger Dana, Mr. Fred Guigliano, Dr. Scott Frasier, Mr. Robert Bogusch and Dr. Dennis Knepp of Mission Research Corporation, Dr. Clifford Prettie of Berkeley Research Associates, and Dr. E. J. Fremouw of North West Research Associates.

CONVERSION TABLE

Conversion factors for U.S. Customary to metric (SI) units of measurement

MULTIPLY ⟶	BY ⟶	TO GET
TO GET ⟵	BY ⟵	DIVIDE

angstrom	1.000000 x E -10	meters (m)
atmosphere (normal)	1.01325 x E +2	kilo pascal (kPa)
bar	1.000000 x E +2	kilo pascal (kPa)
barn	1.000000 x E -28	$meter^2$ (m^2)
British thermal unit (thermochemical)	1.054350 x E +3	joule (J)
calorie (thermochemical)	4.184000	joule (J)
cal (thermochemical) / cm^2	4.184000 x E -2	mega joule/m^2 (MJ/m^2)
curie	3.700000 x E +1	*giga becquerel (GBq)
degree (angle)	1.745329 x E -2	radian (rad)
degree Farenheit	$t_K = (t_F + 459.67)/1.8$	degree kelvin (K)
electron volt	1.60219 x E -19	joule (J)
erg	1.000000 x E -7	joule (J)
erg/second	1.000000 x E -7	watt (W)
foot	3.048000 x E -1	meter (m)
foot-pound-force	1.355818	joule (J)
gallon (U.S. liquid)	3.785412 x E -3	$meter^3$ (m^3)
inch	2.540000 x E -2	meter (m)
jerk	1.000000 x E +9	joule (J)
joule/kilogram (J/kg) (radiation dose absorbed)	1.000000	Gray (Gy)
kilotons	4.183	terajoules
kip (1000 lbf)	4.448222 x E +3	newton (N)
kip/$inch^2$ (ksi)	6.894757 x E +3	kilo pascal (kPa)
ktap	1.000000 x E +2	newton-second/m^2 ($N\text{-}s/m^2$)
micron	1.000000 x E -6	meter (m)
mil	2.540000 x E -5	meter (m)
mile (international)	1.609344 x E +3	meter (m)
ounce	2.834952 x E -2	kilogram (kg)
pound-force (lbs avoirdupois)	4.448222	newton (N)
pound-force inch	1.129848 x E [illegible]	newton-meter (N·m)
pound-force/inch	1.751268 x E +2	[illegible] (N/m)
pound-force/$foot^2$	4.788026 x E -2	kilo pascal (kPa)
pound-force/$inch^2$ (psi)	6.894757	kilo pascal (kPa)
pound-mass (lbm avoirdupois)	4.535924 x E -1	kilogram (kg)
pound-mass-$foot^2$ (moment of inertia)	4.214011 x E -2	kilogram-$meter^2$ ($kg \cdot m^2$)
pound-mass/$foot^3$	1.601846 x E +1	kilogram/$meter^3$ (kg/m^3)
rad (radiation dose absorbed)	1.000000 x E -2	**Gray (Gy)
roentgen	2.579760 x E -4	coulomb/kilogram (C/kg)
shake	1.000000 x E -8	second (s)
slug	1.459390 x E +1	kilogram (kg)
torr (mm Hg, 0° C)	1.333220 x E -1	kilo pascal (kPa)

*The becquerel (Bq) is the SI unit of radioactivity; 1 Bq = 1 event/s.
**The Gray (Gy) is the SI unit of absorbed radiation.

TABLE OF CONTENTS

SECTION 1

RADIO WAVE PROPAGATION IN STRUCTURED IONIZATION FOR SATELLITE AND RADAR APPLICATIONS

1.1 INTRODUCTION.

This report is an extension of References 1[1] and 2[2] which developed the algorithms necessary to calculate the signal structure parameters and the generalized power spectrum which represent the simultaneous phase and amplitude effects of propagating electromagnetic waves through media characterized by a structured index of refraction. Those earlier works were restricted to single component power spectrums, did not treat antenna effects with sufficient generality, and did not treat phase only (total electron content) effects adequately for space radar applications. This paper will eliminate these restrictions.

1.2 ENVIRONMENT CHARACTERIZATION.

The first step in any propagation study is to characterize the ionization or equivalently the index of refraction fluctuations of the ionospheric propagation environment. Figure 1-1 illustrates the geometry of a typical satellite link. The index of refraction fluctuations, represented schematically by the curved lines are typically elongated along the magnetic field. The unit vector $\hat{t}$ is parallel to the magnetic field is a slowly varying function of space since the field lines are curved. The $\hat{z}$ axis and the $\hat{z}$ unit vector represent the propagation line of sight. One end of the radio link is at $z = 0$ and the other end is at $z = z_t$. Except where specifically noted, all quantities calculated are at $z = z_t$. The $\hat{r}$ and $\hat{s}$ unit vectors complete, with $\hat{t}$, an orthogonal coordinate system that is used to define the structure. The orientation of $\hat{r}$ and $\hat{s}$ is chosen to best represent any anisotropy of the index of refraction structure about the field line. As with $\hat{t}$, $\hat{r}$ and $\hat{s}$ may be slow functions of position. In the $\hat{r}$, $\hat{s}$, and $\hat{t}$ system, the index of refraction fluctuations are represented by a power law power spectral density.

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[1] Wittwer, L. A., *Radio Wave Propagation in Structured Ionization for Satellite Applications*, DNA 5304D, 31 December 1979.

[2] Wittwer, L. A., *Radio Wave Propagation in Structured Ionization for Satellite Applications II*, DNA-IR-82-02, 1 August 1982.

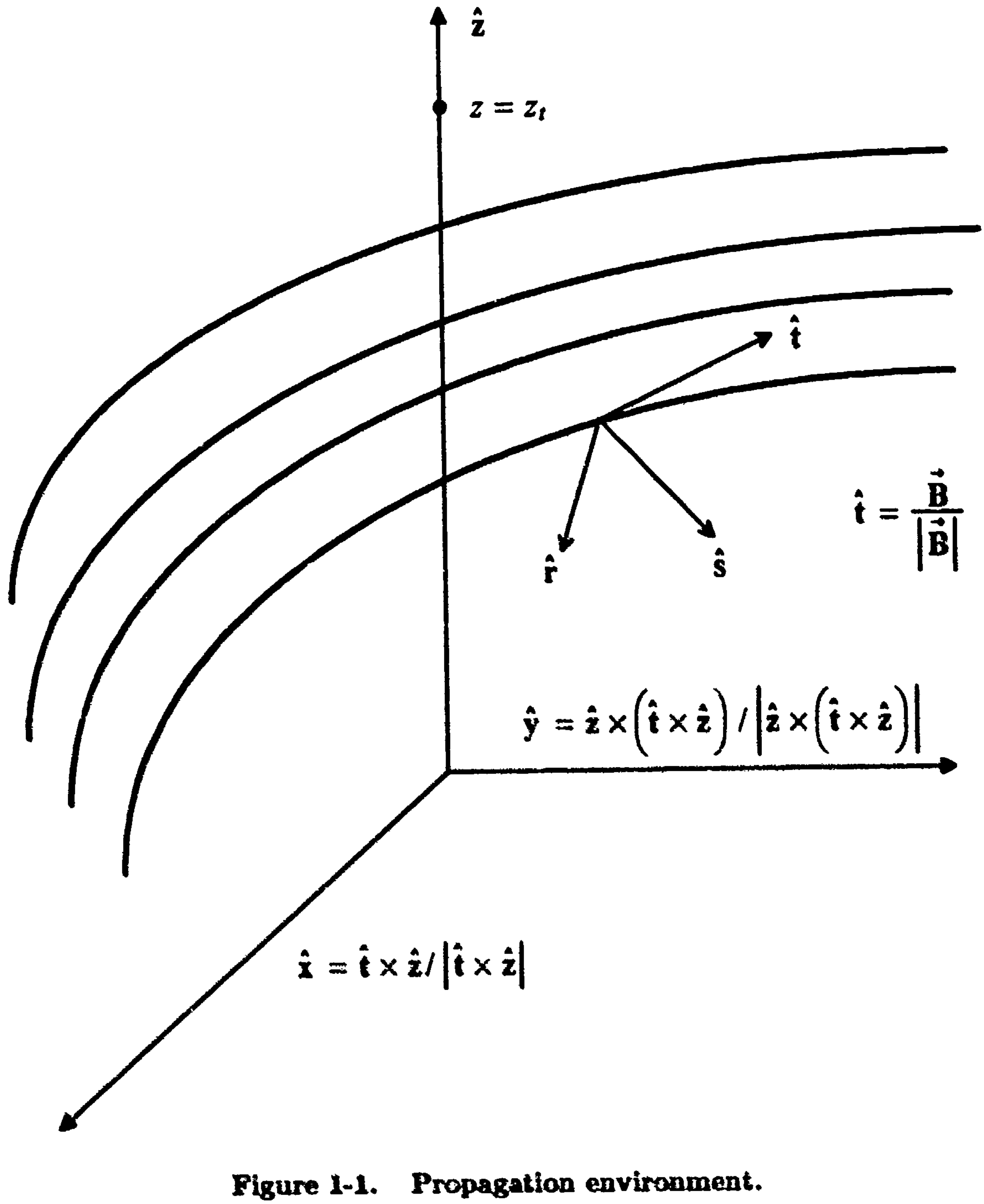

Figure 1-1. Propagation environment.

$$\mathrm{PSD}(K_r, K_s, K_t) = \frac{8\pi^{3/2}\overline{\Delta n_i^2} N_3(n, n', R) L_r L_s L_t \Gamma(n)/\Gamma(n - 3/2)}{(1 + K_r^2 L_r^2 + K_s^2 L_s^2 + K_t^2 L_t^2)^n (1 + K_r^2 \ell_r^2 + K_s^2 \ell_s^2 + K_t^2 \ell_t^2)^{n'-n}} \tag{1.1}$$

where

L_r, L_s, L_t = structure outer scales

ℓ_r, ℓ_s, ℓ_t = structure freezing scales

$\overline{\Delta n_i^2}$ = index of refraction variance

$\Gamma(n)$ = gamma function of argument n

$2n - 2$ = intermediate scale spectral index $(1.5 \leq n \leq 2)$

$2n' - 2$ = transition scale spectral index $(2 \leq n' \leq 4)$

It is assumed that

$$\frac{\ell_r}{L_r} = \frac{\ell_s}{L_s} = \frac{\ell_t}{L_t} = R \leq 1$$

These assumptions regarding the $\hat{r}$, $\hat{s}$ and $\hat{t}$ axes greatly simplify the following development without significantly limiting the applicability of Equation 1.1. It is not claimed that Equation 1.1 with R invariant fully represents all aspects of plasma or other structure. Indeed, it does not. It does, however, provide an adequate representation for those specific features that impact the distortion of radio or radar signals. Equation 1.1 has also found application in specifying infrared and other clutter backgrounds. With the above assumption

$$N_3(n, n', R) = \left[1 - \left(\frac{n' - n}{n' - 1.5}\right)(R f_3(n, n', R))^{2n-3}\right]^{-1}$$

where $f_3(n, n', R)$ is defined in Appendix E in Subroutine PROP.

The structure variation of the index of refraction perpendicular to the $\hat{z}$ axis dominates the propagation effects while the variation parallel to the $\hat{z}$ axis enters only through the strength of the integrated phase variance. Thus Equation 1.1 must be transformed to a frame with one axis being the $\hat{z}$ axis. These new axes are defined by:

$$\hat{x} = \frac{\hat{t} \times \hat{z}}{|\hat{t} \times \hat{z}|} \tag{1.2a}$$

$$\hat{y} = \frac{\hat{z} \times (\hat{t} \times \hat{z})}{|\hat{z} \times (\hat{t} \times \hat{z})|} \tag{1.2b}$$

Two rotations suffice to accomplish the transformation. First, the $\hat{r}$ axis is rotated about the $\hat{t}$ axis by angle ϕ to become parallel to the $\hat{x}$ axis. The angle ϕ is defined by

$$\hat{t} \sin\phi = \frac{\hat{r} \times (\hat{t} \times \hat{z})}{|\hat{t} \times \hat{z}|} \tag{1.3a}$$

$$\cos\phi = \frac{\hat{r} \cdot (\hat{t} \times \hat{z})}{|\hat{t} \times \hat{z}|} \tag{1.3b}$$

Next, the $\hat{t}$ axis is rotated about the $\hat{x}$ axis by angle Φ into the $\hat{z}$ axis. The angle Φ is defined by

$$\hat{x} \sin\Phi = \hat{t} \times \hat{z} \tag{1.4a}$$

$$\cos\Phi = \hat{t} \cdot \hat{z} \tag{1.4b}$$

These transformations can be simplified by defining new effective scale sizes.

$$L_x^2 = L_r^2 \cos^2\phi + L_s^2 \sin^2\phi \tag{1.5a}$$

$$L_y^2 = (L_r^2 \sin^2\phi + L_s^2 \cos^2\phi)\cos^2\Phi + L_t^2 \sin^2\Phi \tag{1.5b}$$

$$L_z^2 = (L_r^2 \sin^2\phi + L_s^2 \cos^2\phi)\sin^2\Phi + L_t^2 \cos^2\Phi \tag{1.5c}$$

$$L_{xy} = (L_s^2 - L_r^2)\cos\Phi\cos\phi\sin\phi \tag{1.5d}$$

$$L_{xz} = (L_r^2 - L_s^2)\sin\Phi\cos\phi\sin\phi \tag{1.5e}$$

$$L_{yz} = (L_t^2 - L_r^2 \sin^2\phi - L_s^2 \cos^2\phi)\cos\Phi\sin\Phi \tag{1.5f}$$

Similar transformations apply to the freezing and inner scales. The final result in the $\hat{x}$, $\hat{y}$, $\hat{z}$ coordinate system is

$$\mathrm{PSD}(K_x, K_y, K_z) = 8\pi^{3/2}\overline{\Delta n_i^2} N_3(n, n', R) L_r L_s L_t \Gamma(n)/\Gamma(n-3/2)\ /$$

$$\Big[(1 + K_x^2 L_x^2 + K_y^2 L_y^2 + K_z^2 L_z^2 + 2L_{xy} K_x K_y + 2L_{xz} K_x K_z + 2L_{yz} K_y K_z)^n$$

$$(1 + K_x^2 \ell_x^2 + K_y^2 \ell_y^2 + K_z^2 \ell_z^2 + 2\ell_{xy} K_x K_y + 2\ell_{xz} K_x K_z + 2\ell_{yz} K_y K_z)^{n'-n}\Big] \quad (1.6)$$

where the small ℓ parameters are the transformed freezing scales The scales ℓ_x and ℓ_y are not signal decorrelation distances in this report.

1.3 DERIVATION OF SIGNAL STRUCTURE PARAMETERS.

The required signal parameters are derived from the differential phase spectrum which is Equation 1.6 with $K_z = 0$ and multiplied by K^2 where K is the carrier frequency wave number. See Appendix D in DNA 5304D. The differential phase spectrum is

$$\frac{dP_\phi(K_x, K_y)}{dz} = 8\pi^{3/2} K^2 \overline{\Delta n_i^2} N_3(n, n', R) L_r L_s L_t \Gamma(n)/\Gamma(n-3/2)\ /$$

$$\Big[(1 + K_x^2 L_x^2 + K_y^2 L_y^2 + 2L_{xy} K_x K_y)^n (1 + K_x^2 \ell_x^2 + K_y^2 \ell_y^2 + 2\ell_{xy} K_x K_y)^{n'-n}\Big] \quad (1.7)$$

The mean square phase fluctuation is

$$\sigma_\phi^2 = \int_0^{z_t} dz\, \frac{d\sigma_\phi^2}{dz} \quad (1.8)$$

where

$$\frac{d\sigma_\phi^2}{dz} = \frac{2\sqrt{\pi}\,\Gamma(n-1) N_3(n, n', R) K^2\, \overline{\Delta n_i^2}\, L_z'}{\Gamma(n-3/2) N_2(n, n', R)} \quad (1.9)$$

$$L_z' = \frac{L_r L_s L_t}{\sqrt{L_x^2 L_y^2 - L_{xy}^2}} \quad (1.10)$$

$$N_2(n, n', R) = \left[1 - \left(\frac{n'-n}{n'-1}\right)(f_2(n, n', R) R)^{2n-2}\right]^{-1}$$

where $f_2(n,n',R)$ is defined in Appendix E in Subroutine PROP. The scale L'_z is the effective z axis scale. Finally

$$\frac{dP_\phi(K_x,K_y)}{dz} = \frac{d\sigma_\phi^2}{dz} 4\pi\sqrt{L_x^2L_y^2 - L_{xy}^2}(n-1)N_2(n,n',R) \quad /$$

$$\left[\left(1+K_x^2L_x^2+K_y^2L_y^2+2L_{xy}K_xK_y\right)^n\left(1+K_x^2\ell_x^2+K_y^2\ell_y^2+2\ell_{xy}K_xK_y\right)^{n'-n}\right] \tag{1.11}$$

In many applications it is necessary to know the one dimensional power spectral density along a fixed direction in the $\hat{x}$, $\hat{y}$ system. Let the direction be defined by the vector, $x_0\,\hat{i} + y_0\,\hat{j}$. The spectrum is approximated by

$$\frac{dP_\phi(K)}{dz} = \frac{d\sigma_\phi^2}{dz}\,\frac{2\sqrt{\pi}\,N_2(n,n',R)\Gamma(n-1/2)(1+6.4/n')\,L}{\Gamma(n-1)(1+6.4/n)\,(1+L^2K^2)^{n-1/2}\,(d^2+\ell^2K^2)^{n'-n}} \tag{1.12}$$

where

$$\begin{aligned}
d^2 &= R^{1.2} + (1-R^{1.2})\left(\frac{1+6.4/n'}{1+6.4/n}\right)^{\frac{1}{n'-n}} \quad , \quad n' \neq n \\
&= R^{1.2} + (1-R^{1.2})\exp\left[\frac{-6.4}{n^2+6.4n}\right] \quad , \quad n' = n \\
L^2 &= \frac{(x_0^2+y_0^2)(L_x^2L_y^2-L_{xy}^2)}{L_x^2y_0^2+L_y^2x_0^2-2L_{xy}x_0y_0} \\
\ell^2 &= R^2L^2 = \frac{(x_0^2+y_0^2)(\ell_x^2\ell_y^2-\ell_{xy}^2)}{\ell_x^2y_0^2+\ell_y^2x_0^2-2\ell_{xy}x_0y_0}
\end{aligned}$$

Let me now define the "local" phase correlation function:

$$R_\phi(\rho(x_0,y_0)) = \left[\frac{1}{2\pi}\int_{-\infty}^{\infty} dK\cos(KL\rho(x_0,y_0))\frac{dP_\phi(K)}{dz}\right] / \frac{d\sigma_\phi^2}{dz}$$

where

$$\begin{aligned}
\rho^2(x_0,y_0) &= \frac{L_x^2y_0^2+L_y^2x_0^2-2L_{xy}x_0y_0}{L_x^2L_y^2-L_{xy}^2} \\
&= \frac{x_0^2+y_0^2}{L^2}
\end{aligned}$$

Finally, let me define the "local" phase structure function:

$$D_\phi(\rho(x_0, y_0)) = 1 - R_\phi(\rho(x_0, y_0)) \tag{1.13a}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} dK \left[2 \sin^2 \left(\frac{K L \rho(x_0, y_0)}{2} \right) \frac{dP_\phi(K)}{dz} \right] / \frac{d\sigma_\phi^2}{dz} \tag{1.13b}$$

Appendix C documents a Fortran function routine to evaluate the phase structure function.

We now consider the calculation of signal parameters with reference to a fixed coordinate system perpendicular to the $\hat{z}$ axis The local $\hat{x}, \hat{y}$ system can rotate in some situations as a function of z. Let us define the $\hat{u}$ and $\hat{v}$ axis perpendicular to the $\hat{z}$. At each point on t'.: $\hat{z}$ axis, the rotation angle between the $\hat{x}$ and $\hat{u}$ axes is defined by

$$\hat{z} \sin\theta = \hat{x} \times \hat{u}$$

$$\cos\theta = \hat{x} \cdot \hat{u}$$

In terms of the $\hat{u}, \hat{v}$ system

$$\rho^2(u_0, v_0) = \frac{L_v^2 u_0^2 + L_u^2 v_0^2 - 2L_{uv} u_0 v_0}{L_u^2 L_v^2 - L_{uv}^2}$$

where

$$L_u^2 = L_x^2 \cos^2\theta + L_y^2 \sin^2\theta + 2L_{xy} \cos\theta \sin\theta$$

$$L_v^2 = L_x^2 \sin^2\theta + L_y^2 \cos^2\theta - 2L_{xy} \cos\theta \sin\theta$$

$$L_{uv} = (L_y^2 - L_x^2) \sin\theta \cos\theta + L_{xy}(\cos^2\theta - \sin^2\theta)$$

The local u_0, v_0 coordinates along the line of sight are

$$u_0 = u \left(\frac{z}{z_t} \right) + u' \left(\frac{z_t - z}{z_t} \right) - \hat{u} \cdot \vec{V}(z)t$$

$$v_0 = v \left(\frac{z}{z_t} \right) + v' \left(\frac{z_t - z}{z_t} \right) - \hat{v} \cdot \vec{V}(z)t$$

where

$$\vec{V} = -\left(\frac{z_t - z}{z_t}\right)\vec{V}_{\rm LOS}(0) + \vec{V}_{\rm ST} - \left(\frac{z}{z_t}\right)\vec{V}_{\rm LOS}(z_t)$$

$$\vec{V}_{\rm LOS}(z) = \text{line of sight velocity}$$

$$\vec{V}_{\rm ST}(z) = \text{structure velocity}$$

The primed coordinates are at $z = 0$ and the unprimed coordinates are at $z = z_t$.

The complex signal space-time correlation function is

$$R(u,v,u',v',t) = \exp\left\{-\int_0^{z_t} dz\,\frac{d\sigma_\phi^2}{dz} D_\phi\left[\rho\left(u_o(u,u',t), v_o(v,v',t)\right)\right]\right\} \tag{1.14}$$

For most applications this function will have the form

$$R(u,v,u',v') = \exp\left\{-\left[\frac{u^2}{\ell_u^2} + \frac{v^2}{\ell_v^2} + \frac{t^2}{\tau_0^2} - 2C_{uv}\frac{uv}{\ell_u\ell_v} - 2C_{ut}\frac{ut}{\ell_u\tau_0} - 2C_{vt}\frac{vt}{\ell_v\tau_0}\right.\right.$$
$$+\frac{u'^2}{\ell_{u'}^2} + \frac{v'^2}{\ell_{v'}^2} - 2C_{u'v'}\frac{u'v'}{\ell_{u'}\ell_{v'}} - 2C_{uu'}\frac{uu'}{\ell_u\ell_{u'}} - 2C_{vv'}\frac{vv'}{\ell_v\ell_{v'}}$$
$$\left.\left. - \text{similar cross terms}\right]\right\} \tag{1.15}$$

where only the most important terms have been written explicitly. For convenience, let

$$I(u,v,u',v',t) = \int_0^{z_t} dz\frac{d\sigma_\phi^2}{dz} D_\phi\left[\rho\left(u_o(u,u',t), v_o(v,v',t)\right)\right]$$

Now ℓ_u, ℓ_v, $\ell_{u'}$, and $\ell_{v'}$ and τ_0 are found by solving

$$I(\ell_u,0,0,0,0) = C(\sigma_\phi^2)$$
$$I(0,\ell_v,0,0,0) = C(\sigma_\phi^2)$$
$$I(0,0,\ell_{u'},0,0) = C(\sigma_\phi^2)$$
$$I(0,0,0,\ell_{v'},0) = C(\sigma_\phi^2)$$
$$I(0,0,0,0,\tau_0) = C(\sigma_\phi^2)$$

The cross terms require a different procedure. For example, consider C_{uv}.

- Solve $I(a_1\ell_u, a_1\ell_v, 0, 0, 0) = C(\sigma_\phi^2)$.
- Let $C_{uv} = 1 - C(\sigma_\phi^2)/(2a_1^2)$.
- If $C_{uv} \geq -0.26$, then you are finished.
- Otherwise, solve $I(-a_1\ell_u, a_1\ell_v, 0, 0, 0) = C(\sigma_\phi^2)$.
- Then $C_{uv} = -1 + C(\sigma_\phi^2)/(2a_1^2)$.

where

$$C(\sigma_\phi^2) = -\ln\left[e^{-1} + exp\left(-\sigma_\phi^2\right) \cdot \left(1 - e^{-1}\right)\right]$$

For defining "reasonable worst case" parameters, we seek the minimum decorrelation lengths at $z = 0$ and $z = z_t$. At $z = z_t$, define ϵ

$$\epsilon = \frac{1}{2}\tan^{-1}\left[\frac{2C_{uv}}{\left(\frac{\ell_u}{\ell_v} - \frac{\ell_v}{\ell_u}\right)}\right]$$

If $\ell_u > \ell_v$, then let $\epsilon = \epsilon + \pi/2$.This angle defines a rotated coordinate system with

$$\begin{aligned} \hat{u} \times \hat{p} &= \hat{z}\sin\epsilon \\ \hat{u} \cdot \hat{p} &= \cos\epsilon \end{aligned}$$

$$\begin{aligned} p &= u\cos\epsilon + v\sin\epsilon \\ q &= -u\sin\epsilon + v\cos\epsilon \end{aligned}$$

The new correlation quantities of dominant concern are

$$\frac{1}{\ell_p^2} = \frac{1}{\ell_u^2}\cos^2\epsilon - \frac{2C_{uv}}{\ell_u\ell_v}\cos\epsilon\sin\epsilon + \frac{1}{\ell_v^2}\sin^2\epsilon \qquad (1.16a)$$

$$\frac{1}{\ell_q^2} = \frac{1}{\ell_u^2}\sin^2\epsilon + \frac{2C_{uv}}{\ell_u\ell_v}\cos\epsilon\sin\epsilon + \frac{1}{\ell_v^2}\cos^2\epsilon \tag{1.16b}$$

$$C_{pq} = 0 \tag{1.16c}$$

$$C_{pt} = \ell_p\left(\frac{C_{ut}}{\ell_u}\cos\epsilon + \frac{C_{vt}}{\ell_v}\sin\epsilon\right) \tag{1.16d}$$

$$C_{qt} = \ell_q\left(\frac{C_{vt}}{\ell_v}\cos\epsilon - \frac{C_{ut}}{\ell_u}\sin\epsilon\right) \tag{1.16e}$$

The energy angle-of-arrival variances are

$$\sigma_{\theta_p}^2 = \frac{2}{K^2\ell_p^2}$$

$$\sigma_{\theta_q}^2 = \frac{2}{K^2\ell_q^2}$$

$\sigma_{\theta_p}^2$ will always be greater than or equal to $\sigma_{\theta_q}^2$, so it is the "reasonable worst case" value. The probability density function for arrival angles is proportional to

$$P(\theta_p,\theta_q) = \frac{1}{2\pi\sigma_{\theta_p}\sigma_{\theta_q}}\exp\left[-\left(\frac{\theta_p^2}{2\sigma_{\theta_p}^2} + \frac{\theta_q^2}{2\sigma_{\theta_q}^2}\right)\right] \tag{1.17}$$

or in "wave number" space

$$P(K_p,K_q) = \pi\ell_p\ell_q\exp\left[-\frac{1}{4}\left(K_p^2\ell_p^2 + K_q^2\ell_q^2\right)\right]$$

The same general set of procedures for calculating "reasonable worst case" parameters must be executed at $z = 0$ since the spatial parameters are not, in general, reciprocal. At the transmitter ($z = 0$), we are generally content with calculating $\ell_{p'}$ and $\ell_{q'}$ (or $\sigma_{\theta_{p'}}^2$ and $\sigma_{\theta_{q'}}^2$).

At the receiver, the signal correlation function is written as

$$R(p,q,t) = \exp\left\{-\left[\frac{p^2}{\ell_p^2} + \frac{q^2}{\ell_q^2} + \frac{t^2}{\tau_0^2} - 2C_{pt}\frac{pt}{\ell_p\tau_0} - 2C_{qt}\frac{qt}{\ell_q\tau_0}\right]\right\}$$

The corresponding power spectrum is

$$F(K_p, K_q, f) = \frac{\pi^{3/2}\ell_p\ell_q\tau_0}{N_0} \exp\Bigg\{-\frac{1}{4N_0^2}\Big[K_p^2\ell_p^2(1 - C_{qt}^2) + K_q^2\ell_q^2(1 - C_{pt}^2) +$$

$$(2\pi\tau_0 f)^2 + 2K_p\ell_pK_q\ell_qC_{pt}C_{qt} - 2K_p\ell_p(2\pi\tau_0 f)C_{pt} - 2K_q\ell_q(2\pi\tau_0 f)C_{qt}\Big]\Bigg\}$$

$$(1.18)$$

where $N_0 = \sqrt{1 - C_{pt}^2 - C_{qt}^2}$.

For radar application, it is necessary to know the decorrelation between the outward path to the target and the return path. Assuming that the radar is at $z = z_t$, the two way decorrelation is

$$\mathrm{TWPD} = \exp\left\{-\int_0^{z_t} dz \, \frac{d\sigma_\phi^2}{dz} \, D_\phi \left[\frac{2z}{c}\sqrt{\frac{L_y^2V_{TX}^2 + L_z^2V_{TY}^2 - 2L_{zy}V_{TX}V_{TY}}{L_z^2L_y^2 - L_{zy}^2}}\right]\right\} \quad (1.19)$$

where

$$\begin{aligned} c &= \text{speed of light} = 3 \times 10^8 \text{m/sec} \\ V_{TX} &= \vec{V}_T(z) \cdot \hat{x} \\ V_{TY} &= \vec{V}_T(z) \cdot \hat{y} \\ \vec{V}_T(z) &= \vec{V}_{ST}(z) - \left(\frac{z}{z_t}\right)\vec{V}_{LOS}(z_t) \end{aligned}$$

We define the parallel signal decorrelation time as

$$\tau_{\parallel} = \frac{2\pi}{K\left[\,|(\vec{V}_{z=0} \cdot \hat{z}) - V_z|\sigma_\phi'^2 + |(\vec{V}_{z=t} \cdot \hat{z}) - V_z|\sigma_\phi^2\,\right]}$$

$$V_z = \frac{1}{\sigma_\phi^2}\int_0^{z_t} dz \, \frac{d\sigma_\phi^2}{dz} \, (\vec{V} \cdot \hat{z})$$

The final decorrelation time is

$$\tau_0 = \min(\tau_0, \tau_{\parallel}) \tag{1.20}$$

where the τ_0 within the parentheses was calculated earlier.

The next quantity required is the frequency selective bandwidth, f_0.

$$f_0 = \frac{f_2'}{\sqrt{1 + C_1^2}} \tag{1.21}$$

where C_1 = delay parameter ($\approx$0.25) and

$$\frac{1}{f_2'^2} = \frac{4\pi^2}{K^4 c^2} \int_0^{z_t} \frac{dz'}{z'^2} \int_0^{z'} \frac{dz}{z^2} \left[I_u^2(z) + I_v^2(z) + 2I_{uv}^2(z) \right]$$

$$\begin{aligned}
I_u(z') &= \int_0^{z'} dz \, \frac{d\sigma_\phi^2}{dz} \, B_2 \left(\frac{2z^2 L_v^2}{L_x^2 L_y^2 - L_{xy}^2} \right) \\
I_v(z') &= \int_0^{z'} dz \, \frac{d\sigma_\phi^2}{dz} \, B_2 \left(\frac{2z^2 L_u^2}{L_x^2 L_y^2 - L_{xy}^2} \right) \\
I_{uv}(z') &= \int_0^{z'} dz \, \frac{d\sigma_\phi^2}{dz} \, B_2 \left(\frac{2z^2 L_{uv}}{L_x^2 L_y^2 - L_{xy}^2} \right)
\end{aligned}$$

Appendix D discusses the derivation of the above equation. Also

$$\begin{aligned}
L_u^2 &= L_x^2 \cos^2\theta + L_y^2 \sin^2\theta + 2L_{xy} \sin\theta \cos\theta \\
L_v^2 &= L_x^2 \sin^2\theta + L_y^2 \cos^2\theta - 2L_{xy} \sin\theta \cos\theta \\
L_{uv} &= (L_y^2 - L_x^2) \sin\theta \cos\theta + L_{xy}(\cos^2\theta - \sin^2\theta) \\
\ell_1 &= \left[\frac{1}{\ell_u^4} + \frac{1}{\ell_v^4} + 2\left(\frac{C_{uv}}{\ell_u \ell_v} \right)^2 \right]^{-1/4} \\
\ell_2 &= \left[\frac{1}{\ell_{u'}^4} + \frac{1}{\ell_{v'}^4} + 2\left(\frac{C_{u'v'}}{\ell_{u'} \ell_{v'}} \right)^2 \right]^{-1/4} \\
L &= \left[\frac{L_v^4 + L_u^4 + 2L_{uv}^2}{\left(L_x^2 L_y^2 - L_{xy}^2 \right)^2} \right]^{-1/4}
\end{aligned}$$

$$\rho = \frac{\sqrt{\ell_1 \ell_2 z(z_t - z)}}{z_t L}$$

$$B_2 = \frac{D_\phi(\rho)}{\rho^2}$$

The final generalized power spectrum at the receiver is usually expressed as

$$\Gamma(K_p, K_q, f, \tau) = 2\pi f_0 F(K_p, K_q, f)\, \delta \left[2\pi f_0 \tau - \frac{\sqrt{2}\, \ell_p^2 \ell_q^2}{\sqrt{\ell_p^4 + \ell_q^4}} \frac{(K_p^2 + K_q^2)}{4} \right] \tag{1.22}$$

where

$$\int_{-\infty}^{\infty} dx\, \delta(x-a) f(x) = f(a)$$

$$\int_{-\infty}^{\infty} dx\, \delta(x-a) = 1$$

The mean square log amplitude fluctuation is used to denote the presence of amplitude fluctuations. It is

$$\overline{\chi^2} = \int_0^{z_t} dz\, \frac{d\sigma_\phi^2}{dz}\, X2(z) \tag{1.23a}$$

$$X2(z) = N_2(n, n', R)(n-1) \int_0^{\infty} \frac{.5[1 - \cos(M_a y) J_0(M_b y)]}{(1+y)^n (1 + R^2 y)^{n'-n}} dy \tag{1.23b}$$

where

$$M_a = \frac{z(z_t - z)}{K z_t} \cdot \frac{L_x^2 + L_y^2}{2(L_x^2 L_y^2 - L_{xy}^2)}$$

$$M_b = \frac{z(z_t - z)}{K z_t} \cdot \frac{\sqrt{(L_y^2 - L_x^2)^2 + 4 L_{xy}^2}}{2(L_x^2 L_y^2 - L_{xy}^2)}$$

Appendix A discusses the development of Equation 1.23 and presents an algorithm to evaluate Equation 1.23b.

The $\overline{\chi^2}$ algorithm is accurate to better than ten percent. The quantity $\overline{\chi^2}$ should henceforth be carried as a primary quantity to measure amplitude fluctuations. This function was previously done by $\sigma^2_{\phi R}$, the Rayleigh phase fluctuation parameter.

1.4 EFFECTIVE SCALE SIZES AND INDICES.

For most applications, the effective scales and indices are used to calculate the statistics of the environment produced Doppler, Doppler rate, group delay, etc. At each position along the line of sight, we define the scales along the local velocity vector. Then

$$\begin{aligned} L^2(z) &= \frac{(V_x^2(z)+V_y^2(z))L_x^2L_y^2}{V_x^2(z)L_y^2+V_y^2(z)L_x^2} \\ \ell^2(z) &= \frac{(V_x^2(z)+V_y^2(z))\ell_x^2\ell_y^2}{V_x^2(z)\ell_y^2+V_y^2(z)\ell_x^2} \\ &= R^2L^2 \end{aligned}$$

Let

$$G(m) = \left(\frac{|\vec{V}|}{L}\right)^m \left\{\frac{1}{m+1} + R^{2n-m-2}\left[F(m+2-2n,R) + F(2n'-m-2,R')\right]\right\}$$

$$\begin{aligned} R' &= \ell_i/\ell \quad &, \quad & \ell_i = \text{inner scale} \\ F(n,R) &= \tfrac{1}{n}(1-R^n) \quad &, \quad & n \neq 0 \\ &= \ln(1/R) \quad &, \quad & n = 0 \\ |\vec{V}| &= \sqrt{V_x^2(z)+V_y^2(z)} \end{aligned}$$

$$I_0 = \int_0^{z_t} dz\, \frac{d\sigma_\phi^2}{dz}\, \frac{G(2)}{G(0)}$$

Finally

$$L_{\text{eff}} = \left[\frac{1}{I_0}\int_0^{z_t} dz\, \frac{d\sigma_\phi^2}{dz}\, \frac{G(2)}{G(0)}\, \frac{L}{|\vec{V}|}\right] V_{\text{eff}} \tag{1.24a}$$

$$\ell_{\text{eff}} = \left[\frac{1}{I_0}\int_0^{z_t} dz\,\frac{d\sigma_\phi^2}{dz}\,\frac{G(2)}{G(0)}\,\frac{\ell}{|\vec{V}|}\right] V_{\text{eff}} \tag{1.24b}$$

$$\ell_{\text{ieff}} = \left[\frac{1}{I_0}\int_0^{z_t} dz\,\frac{d\sigma_\phi^2}{dz}\,\frac{G(2)}{G(0)}\,\frac{\ell_i}{|\vec{V}|}\right] V_{\text{eff}} \tag{1.24c}$$

$$n_{\text{eff}} = \frac{1}{I_0}\int_0^{z_t} dz\,\frac{d\sigma_\phi^2}{dz}\,\frac{G(2)}{G(0)}\,n \tag{1.24d}$$

$$n'_{\text{eff}} = \frac{1}{I_0}\int_0^{z_t} dz\,\frac{d\sigma_\phi^2}{dz}\,\frac{G(2)}{G(0)}\,n' \tag{1.24e}$$

At this point, σ_ϕ^2 must be redefined to reflect the "Doppler power." Let

$$\begin{aligned}
R &= \ell_{\text{eff}}/L_{\text{eff}} \\
R' &= \ell'_{\text{eff}}/\ell_{\text{eff}} \\
n &= n_{\text{eff}} \\
n' &= n'_{\text{eff}} \\
|\vec{V}| &= V_{\text{eff}} = \ell''/\tau_0 \\
L &= L_{\text{eff}}
\end{aligned}$$

$$\int_0^{z_t} dz\,\frac{d\sigma_\phi^2}{dz}\,D\left(\frac{\ell''}{L(z)}\right) = C(\sigma_\phi^2)$$

in the $G(m)$ function. Then

$$\sigma_\phi^2 = I_0\,\frac{G(0)}{G(2)} \tag{1.25}$$

For most applications, V_{eff} can be approximated by

$$V_{\text{eff}} = \frac{\ell_p}{\tau_0}$$

1.5 SUMMARY.

The preceeding sections extended the results of References 1 and 2 to permit the prediction of the effects of structured index of refraction on electromagnetic waves to two component power spectrums. Six new parameters, σ_ϕ^2, L_{eff}, and ℓ_{eff}, ℓ_{ieff}, n_{eff} and n'_{eff}, were introduced to describe the total electron content power spectrum. One new parameter was introduced for some radar applications. Finally, the Rayleigh phase variance has been eliminated in favor of the mean square log amplitude fluctuation for indicating amplitude fluctuations. Appendix E documents a FORTRAN subroutine which calculates the described signal parameters.

APPENDIX A

CALCULATION OF THE MEAN SQUARE LOG AMPLITUDE FLUCTUATION

The equation for the mean square log amplitude fluctuation is

$$\overline{\chi^2} = \int_0^{z_t} dz \int_{-\infty}^{\infty} \frac{dK_x}{2\pi} \int_{-\infty}^{\infty} \frac{dK_y}{2\pi} \sin^2\left[\frac{(K_x^2 + K_y^2)(z_t - z)z}{2Kz_t}\right] \frac{dP_\phi(K_x, K_y)}{dz} \qquad \text{(A.1)}$$

where $\frac{dP_\phi(K_x, K_y)}{dz}$ is found in Equation 1.11. Equation A.1 can be transformed (as shown by Scott Frasier) into

$$\overline{\chi^2} = \int_0^{z_t} dz\, N_2(n, n', R)(n-1)\frac{d\sigma_\phi^2}{dz} \int_0^{\infty} \frac{.5[1 - \cos(M_a(z)y))J_0(M_b(z)y)]}{(1+y)^n(1+R^2y)^{n'-n}}\, dy \qquad \text{(A.2)}$$

where

$$M_a(z) = \frac{z(z_t - z)}{Kz_t} \frac{L_x^2 + L_y^2}{2(L_x^2L_y^2 - L_{xy}^2)}$$

$$M_b(z) = \frac{z(z_t - z)}{Kz_t} \frac{\sqrt{(L_y^2 - L_x^2)^2 + 4L_{xy}^2}}{2(L_x^2L_y^2 - L_{xy}^2)}$$

To avoid the Bessel function and the cosine, the numerator in the inner integral is approximated by

$$.5[1 - \cos(M_a(z)y))J_0(M_b(z)y)] = \begin{cases} a^2 & , \quad a < \sqrt{c_1} \\ c_1\left(\frac{a}{\sqrt{c_1}}\right)^b & , \quad \sqrt{c_1} \le a \le 3.1 \\ 0.5 & , \quad 3.1 \le a \end{cases}$$

where

$$
\begin{aligned}
a &= M(z)y \\
M(z) &= \sqrt{\frac{2M_a^2(z) + M_b^2(z)}{8}} \\
b &= \ln(0.5/c_1)/\ln(3.1/\sqrt{c_1}) \\
c_1 &= \frac{(1 + R_t/2)^2}{1.2 + 4.8R_t} \\
R_t &= M_b^2/M_a^2
\end{aligned}
$$

Equation A.2 using the above approximation can be rapidly integrated since the integrand is power law over long ranges of y. Points can be chosen to closely approximate the integrand as a series of power law segments. Each segment can be analytically integrated. Appendix E shows a line of sight integrator that uses this method.

APPENDIX B

LARGE TARGET, LARGE APERTURE SIGNAL STRUCTURES

The signal at coordinates $(\vec{g}_1, z)$ received from coordinates $(\vec{h}_1, \phi)$ can be written as

$$E(z, \vec{g}_1, \vec{h}_1, K_1) = U(z, \vec{g}_1, \vec{h}_1, K_1) \frac{\exp\left(iK_1\sqrt{z^2 + |\vec{g}_1 - \vec{h}_1|^2}\right)}{\sqrt{z^2 + |\vec{g}_1 - \vec{h}_1|^2}} \tag{B.1}$$

where

$$\begin{aligned} \vec{g}_1 &= x_1\,\hat{\imath} + y_1\,\hat{\jmath} \\ \vec{h}_1 &= u_1\,\hat{\imath} + v_1\,\hat{\jmath} \end{aligned}$$

Applying the parabolic approximation

$$E(z, \vec{g}_1, \vec{h}_1, K_1) = U(z, \vec{g}_1, \vec{h}_1, K_1) \frac{\exp(iK_1 z)\exp\left[\frac{iK_1|\vec{g}_1 - \vec{h}_1|^2}{2z}\right]}{z} \tag{B.2}$$

The autocorrelation function can be written as

$$\begin{aligned} \overline{E^*(z, \vec{g}_1, \vec{h}_1, K_1)E(z, \vec{g}_1, \vec{h}_1, K_1)} &= \frac{G(z, \vec{g}, \vec{G}, \vec{h}, \vec{H}, \Delta k, K)}{z^2} \\ &\times \exp\left\{\frac{iK}{z}\left[\vec{H}\cdot\vec{h} + \vec{G}\cdot\vec{g} - \vec{g}\cdot\vec{H} - \vec{G}\cdot\vec{h}\right]\right. \\ &\left. + \frac{i\Delta k}{z}\left[\frac{(\vec{H}\cdot\vec{H} + \vec{G}\cdot\vec{G})}{2} + \frac{(\vec{h}\cdot\vec{h} + \vec{g}\cdot\vec{g})}{8} - \vec{H}\cdot\vec{G} - \frac{\vec{h}\cdot\vec{g}}{4}\right]\right\} \end{aligned} \tag{B.3}$$

where

$$G(z, \vec{g}, \vec{G}, \vec{h}, \vec{H}, \Delta k, K) = \overline{U^*(z, \vec{g}_1, \vec{h}_1, K_1)U(z, \vec{g}_2, \vec{h}_2, K_2)}$$

$$\vec{G} = \frac{\vec{g}_1 + \vec{g}_2}{2} = X\,\hat{\imath} + Y\,\hat{\jmath}$$

$$X = \frac{x_1 + x_2}{2} \quad , \quad Y = \frac{y_1 + y_2}{2}$$

$$\vec{g} = \vec{g}_2 - \vec{g}_1 = x\,\hat{\imath} + y\,\hat{\jmath}$$

$$x = x_2 - x_1 \quad , \quad y = y_2 - y_1$$

$$\vec{H} = \frac{\vec{h}_1 + \vec{h}_2}{2} = U\,\hat{\imath} + V\,\hat{\jmath}$$

$$U = \frac{u_1 + u_2}{2} \quad , \quad V = \frac{v_1 + v_2}{2}$$

$$\vec{h} = \vec{h}_2 - \vec{h}_1 = u\hat{\imath} + v\hat{\jmath}$$

$$u = u_2 - u_1 \quad , \quad v = v_2 - v_1$$

$$\begin{aligned} \Delta k &= K_2 - K_1 \\ K &= \frac{K_1 + K_2}{2} \end{aligned}$$

The equation for $G(z, \vec{g}, \vec{G}, \vec{h}, \vec{H}, \Delta k, K)$ is

$$\frac{\partial G}{\partial z} = -\frac{i\Delta k}{2K_1K_2}\left[\frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2} + \frac{1}{4}\left(\frac{\partial^2 G}{\partial X^2} + \frac{\partial^2 G}{\partial Y^2}\right)\right] + \frac{iK}{K_1K_2}\left[\frac{\partial^2 G}{\partial x \partial X} + \frac{\partial^2 G}{\partial y \partial Y}\right]$$
$$-\frac{(x-u)}{z}\frac{\partial G}{\partial x} - \frac{(X-U)}{z}\frac{\partial G}{\partial X} - \frac{(y-v)}{z}\frac{\partial G}{\partial y} - \frac{(Y-V)}{z}\frac{\partial G}{\partial Y}$$
$$-\frac{1}{2}\left[\frac{K_1^2+K_2^2}{K_1K_2}\right]\frac{d\sigma_\phi^2}{dz}G + \frac{d\sigma_\phi^2}{dz}R_\phi(x - V_xt, X, y - V_yt, Y, z)G \tag{B.4}$$

where $\frac{d\sigma_\phi^2}{dz}$ is evaluated for a wavenumber of $\sqrt{K_1K_2}$. Also, V_x and V_y have been introduced to represent the motion of the environment with respect to the line of sight. The mixing of the notation for the wavenumbers was for convenience. R_ϕ is the normalized integrated phase autocorrelation function.

$$R_\phi(0,0,0,0,z) = 1$$

V_x and V_y are local velocities and t is the time displacement. The exponential term divided by z^2 in Equation B.3 contains the free space solution for the total signal autocorrelation function. Thus in the absence of any signal scattering, G is always equal to one. In general, R_ϕ does not depend on X or Y. Since G does not have any dependence on X or Y in free space, it can never develop any. Equation B.4 can thus be written

$$\frac{\partial G(z,\vec{g},\vec{h},\Delta k,t)}{\partial z} = -\frac{i\Delta k}{2K_1K_2}\left[\frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2}\right] - \frac{(x-u)}{z}\frac{\partial G}{\partial x} - \frac{(y-v)}{z}\frac{\partial G}{\partial y}$$
$$-\frac{1}{2}\left[\frac{K_1^2+K_2^2}{K_1K_2}\right]\frac{d\sigma_\phi^2}{dz}G + \frac{d\sigma_\phi^2}{dz}R_\phi(x - V_xt, y - V_yt, z)G \tag{B.5}$$

Let

$$D_\phi(\rho(x_0,y_0)) = 1 - R_\phi(x_0,y_0,z)$$

where

$$\rho^2(x_0, y_0) = \frac{L_y^2 x_0^2 + L_x^2 y_0^2 - 2L_{xy} x_0 y_0}{L_x^2 L_y^2 - L_{xy}^2}$$

$$L_x, L_y, L_{xy} = \text{environment outer scales}$$

The z dependence in D_ϕ is implicit in the scale sizes and the exact functional dependence of R_ϕ. D_ϕ is the "local" phase structure function.

Equation B.5 simplifies to

$$\frac{\partial G(z, \vec{g}, \vec{h}, \Delta k, K)}{\partial z} = -\frac{i\Delta k}{2K_1K_2}\left[\frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2}\right] - \frac{(x-u)}{z}\frac{\partial G}{\partial x} - \frac{(y-v)}{z}\frac{\partial G}{\partial y}$$
$$-\frac{\Delta k^2}{2K_1K_2}\frac{d\sigma_\phi^2}{dz}G - \frac{d\sigma_\phi^2}{dz}D_\phi(\rho(x - V_x t, y - V_y t))G \quad \text{(B.6)}$$

Let

$$G = G_1 \exp\left[-\frac{\Delta k^2}{2K_1K_2}\int_0^z dz' \frac{d\sigma_\phi^2}{dz'}\right] \quad \text{(B.7)}$$

$$\theta_x = \frac{x-u}{z}$$

$$\theta_y = \frac{y-v}{z}$$

Then

$$\frac{\partial G_1}{\partial z} = -\frac{i\Delta k}{2K_1K_2z^2}\left[\frac{\partial^2 G_1}{\partial \theta_x^2} + \frac{\partial^2 G_1}{\partial \theta_y^2}\right] - \frac{d\sigma_\phi^2}{dz}D_\phi(\rho(z\theta_x + u - V_x t, z\theta_y + v - V_y t))G_1$$

(B.8)

If frequency selective effects are ignored, then θ_x and θ_y become parameters. Let

$$\theta_x = \frac{x-u}{z_t}$$
$$\theta_y = \frac{y-v}{z_t}$$

where z_t is the point where the receive or target is located. Now

$$\frac{\partial G}{dz} = -\frac{d\sigma_\phi^2}{dz} G_1 D_\phi \left[\rho\left(\frac{zx}{z_t} + \frac{(z_t-z)u}{z_t} - V_x t\,,\ \frac{zy}{z_t} + \frac{(z_t-z)v}{z_t} - V_y t\right)\right] \tag{B.9}$$

In the following let $t = 0$.

The solution to Equation B.9 can often be approximated by

$$G_1(x,u,y,v) = G_1(0,0,0,0)\exp\left\{-\left[\frac{x^2}{\ell_x^2} + \frac{y^2}{\ell_y^2} + \frac{u^2}{\ell_u^2} + \frac{v^2}{\ell_v^2} - 2C_{xu}\frac{xu}{\ell_x\ell_u} - 2C_{yv}\frac{yv}{\ell_y\ell_v} \right.\right.$$
$$\left.\left. -2C_{xy}\frac{xy}{\ell_x\ell_y} - 2C_{xv}\frac{xv}{\ell_x\ell_v} - 2C_{yu}\frac{uy}{\ell_u\ell_y} - 2C_{uv}\frac{uv}{\ell_u\ell_v}\right]\right\} \tag{B.10}$$

The above coefficients are easy to calculate. Let us define the integral

$$I(x,u,y,v) = \int_0^{z_t} dz \frac{d\sigma_\phi^2}{dz} D_\phi \left[\rho\left(\frac{zx}{z_t} + \frac{(z_t-z)u}{z_t}\,,\ \frac{zy}{z_t} + \frac{(z_t-z)v}{z_t}\right)\right] \tag{B.11}$$

Then

$$I(\ell_x,0,0,0) = C(\sigma_\phi^2)$$
$$I(0,\ell_u,0,0) = C(\sigma_\phi^2)$$
$$I(0,0,\ell_y,0) = C(\sigma_\phi^2)$$
$$I(0,0,0,\ell_v) = C(\sigma_\phi^2)$$

where

$$C(\sigma_\phi^2) = -\ln\left[e^{-1} + exp\left(-\sigma_\phi^2\right)\cdot\left(1 - e^{-1}\right)\right]$$

The cross terms are also straightforward. For example, let us calculate C_{xv}. Solve

$$I(a\ell_x, 0, 0, a\ell_v) = C(\sigma_\phi^2)$$

$$C_{xv} = 1 - C(\sigma_\phi^2)/2a^2$$

The other cross terms are similarly calculated.

Equation B.10 is the basic equation for large target, large aperture applications. If the large aperture is a synthetic aperture radar (SAR), it is convenient to place the u axis along the satellite track with $v = 0$. Then

$$G_1(x,u,y) = G_1(0,0,0)\exp\left\{-\left[\frac{x^2}{\ell_x^2} + \frac{y^2}{\ell_y^2} + \frac{u^2}{\ell_u^2} - 2C_{xu}\frac{xu}{\ell_x\ell_u} - 2C_{xy}\frac{xy}{\ell_x\ell_y} - 2C_{yu}\frac{yu}{\ell_y\ell_u}\right]^{m/2}\right\}. \quad \text{(B.12)}$$

For generating of signal structure we need the power spectrum, that is, the Fourier transform of Equation B.12. Assuming $m = 2$, let

$$N_0 = \sqrt{1 - C_{xy}^2 - C_{xu}^2 - C_{yu}^2 - 2C_{yu}^2C_{xy}^2C_{xu}^2}$$

The power spectrum is

$$\tilde{G}_1(K_x, K_y, K_u) = \frac{\pi^{3/2}G_1(0,0,0)\ell_x\ell_y\ell_u}{N_0}\exp\left\{-\frac{1}{4N_0^2}\left[K_x^2\ell_x^2(1 - C_{yu}^2) + K_y^2\ell_y^2(1 - C_{xu}^2) + K_u^2\ell_u^2(1 - C_{xy}^2) + 2K_u\ell_uK_x\ell_x(C_{xu} + C_{yu}C_{yx}) + 2K_u\ell_uK_y\ell_y(C_{yu} + C_{xy}C_{xu}) + 2K_x\ell_xK_y\ell_y(C_{xy} + C_{xu}C_{yu})\right]\right\}$$

(B.13)

In general, the most stressing case for a SAR occurs when the satellite radar beam travels perpendicular with respect to the magnetic field in the scattering medium. In this case

$$C_{xy} \approx C_{yu} \approx 0$$

Then

$$\tilde{G}_1 = \frac{\pi^{3/2} G_1(0,0,0)\ell_x \ell_y \ell_u}{\sqrt{1 - C_{xu}^2}} \exp\left\{-\frac{1}{4}\left[\frac{K_x^2 \ell_x^2 + 2C_{xu} K_x \ell_x K_u \ell_u + K_u^2 \ell_u^2}{1 - C_{xu}^2} + K_y^2 \ell_y^2\right]\right\} \tag{B.14}$$

The y direction is decoupled from the x, u direction. C_{xu} is less than one and relates to the thickness of the scattering medium. In the limit of a delta layer, C_{xu} approaches one and K_u approaches K_x.

APPENDIX C

CALCULATION OF THE "LOCAL" PHASE STRUCTURE FUNCTION

The "local" structure function is calculated with a Fortran call to CNNXM. The formal equivalence is

$$D(\rho(x_o, y_o)) = \mathrm{CNNXM}(\rho(x_o, y_o), IR)$$

IR anticipates the need to iterate over points along the propagation line of sight (LOS) several times in calculating some propagation parameters. IABS(IR) is used as an index over LOS integration points. Arrays internal to CNNXM must be large enough to accommodate the largest number of integration points. See comments inside of the CNNXM source code. On the first call to CNNXM at an integration point, four variables must be provided to CNNXM via the labeled common block, CNDATA. The block definition and the variables are

REAL N,NP

COMMON /CNDATA/N,NP,R,RP

N = n

NP = n'

R = freezing to outer scale size ratio

RP = inner to outer scale size ratio

On the subsequent calls at that point, use the positive index. The data in the labeled common block is ignored. Stored data from the first call is used to provide a fast result.

The following block of code contains a basic iteration loop to calculate a decorrelation distance, $\rho(x_0, y_0)$. This value is at the EXP(-1) point of the complex signal correlation function. This code is easily generalized to multiple integration points. For most cases it is most efficient to begin the iteration with a small value of XS= $\rho(x_0, y_0)$ =RP. The error in the final value for $\rho(x_0, y_0)$ is less than five percent.

```
      PROGRAM TEST
      REAL N,NP,M,LO,LF
      COMMON /CNDATA/N,NP,R,RP
C
C LO = OUTER SCALE
C
      LO=1.0E7
C
C LF = FREEZING SCALE
C
      LF=1.0E5
C
C P2 = MEAN SQUARE PHASE FLUCTUATION
C
      P2=1000.0
C
C SET UP LABELED COMMON BLOCK FOR NEGATIVE INTEGRATION INDEX
C     2*N-2  = INTERMEDIATE SCALE SPECTRAL INDEX
C     2*NP-2 = TRANSITION SCALE SPECTRAL INDEX
C     R      = FREEZING TO OUTER SCALE RATIO
C     RP     = INNER TO OUTER SCALE RATIO
C
      N=1.6
      NP=3.0
      R=LF/LO
      RP=R/100.0
C
C START ITERATION
C
      CLIM=-ALOG(EXP(-1.0)+(1.0-EXP(-1.0))*EXP(-P2))
      XS=RP
      M=2.0
      ANO=CNNXM(XS,-1)
      XO=RP/SQRT(P2*ANO/CLIM)
56    ANSS=CNNXM(XO,1)
      DP=P2*ANSS/CLIM
      IF(ABS(1.0-DP).LT.0.001)GO TO 57
      M=ALOG(ANSS/ANO)/ALOG(XO/XS)
      ANO=ANSS
```

```
      XS=XO
      XO=XO/DP**(1.0/M)
      IF(XO.LT.(XS/10.0))XO=XS/10.0
      GO TO 56
57    CONTINUE
C
C AT THIS POINT THE RESULTS ARE
C     M=EFFECTIVE SPECTRAL INDEX
C     XO=FINAL SOLUTION FOR RHO(X_0,Y_0)
C
```

The following code is the CNNXM Fortran function. The internal arrays must be sized to the largest number of LOS integration points to be used. The size is specified by the internal parameter NIP.

```
      FUNCTION CNNXM(X,IFLG)
C
C FUNCTION CNNXM, VERSION 3.0,  4 APR 89
C
C THIS IS THE PHASE STRUCTURE FUNCTION FOR THE WITTWER-KILB POWER
C SPECTRUM.  WHEN USED TO CALCULATE THE SIGNAL DECORRELATION
C DISTANCE, THE MAXIMUM ERROR IN THE DISTANCE IS FIVE PERCENT
C THE MAXIMUM ERROR IN THE STRUCTURE FUNCTION, ITSELF, IS ABOUT TEN
C PERCENT.
C
C THE INPUT VARIABLES ARE:
C
C   LABELED COMMON BLOCK /CNDATA/
C    (THESE VARIABLES ARE USED ONLY WHEN IFLG < 0)
C      2*N-2 = INTERMEDIATE SCALE SPECTRAL INDEX
C              (N.GE.1.6.AND.N.LE.2.0)
C      2*NP-2 = SMALL SCALE SPECTRAL INDEX
C               (NP.GE.2.0.AND NP.LE.4.0)
C      R = RATIO OF FREEZING TO OUTER SCALE
C          (R.LE.1.AND R.GE.1.0E-07)
C      RP = RATIO INNER TO OUTER SCALE
C           (RP.LE.R)
C
```

```
C   FORMAL ARGUMENTS
C      X = DISPLACEMENT DIVIDED BY OUTER SCALE
C      IABS(IFLG) = INDEX OF LOS INTEGRATION POINT
C                   (IABS(IFLG).LE.NIP.AND.IFLG.NE.0)
C      IFLG < 0, CALCULATE AND STORE INTERMEDIATE RESULTS.  THIS
C                IS USED WHENEVER N, NP, R, OR RP CHANGES FOR
C                SOME VALUE OF IABS(IFLG).
C      IFLG > 0, USE STORED INTERMEDIATE DATA.  ASSUMES THAT
C                N, NP, R, AND RP HAVE NOT CHANGED FOR CURRENT
C                VALUE OF IABS(IFLG) SINCE LAST CALL.
C
      SAVE
      REAL N,NP,L2
C
C   NIP=NUMBER OF LOS INTEGRATION POINTS
C
      PARAMETER (NIP=11,E2=-7.1509297E-01)
      PARAMETER (D3=6.1788469E-01,C3=9.5310179E-02)
      PARAMETER (D4=1.3862943,C1=-6.9314718E-01)
      PARAMETER (E7=2.8490703E-01,C4=6.5551593E-01)
      DIMENSION XPSD(12*NIP),FP(NIP),E4(NIP),JP(NIP)
      DIMENSION E5(NIP),L2(NIP),IA7(NIP),JM(NIP)
      DIMENSION XPSDP(12*NIP),XPSDM(12*NIP),XN(12*NIP)
      COMMON /CNDATA/N,NP,R,RP
      I=IABS(IFLG)
      JSTART=(I-1)*12+1
      IF(IFLG.LT.0)THEN
C
C DO A3=d0**2
C
         A3=R**1.2
         IF(ABS(N-NP).LT.0.001)THEN
            A6=EXP(-6.4/(N*(N+6.4)))
         ELSE
            A6=((1.0+6.4/NP)/(1.0+6.4/N))**(1.0/(NP-N))
         END IF
         A3=A3+(1.0-A3)*A0
C
C INITIAL NORMALIZATION CONSTANT
C
```

```
      A6=1.00+((0.06*NP-0.54)*NP+0.38)*ALOG(R)
      A6=(A6*R)**(2.0*N-2.0)
      A6=1.0/(1.0-(NP-N)*A6/(NP-1.0))
      FP(I)=0.24*A6*(N-1.0)*(1.0+6.4/NP)/A3**(NP-N)
C
C CONSTANTS
C
      L2(I)=R*R/A3
C     X0=ALOG(1.0/SQRT(L2))
      X0=-0.5*ALOG*(L2(I))
      IF(X0.LT.2.05)THEN
          A3=0.951220*X0-1.0
          XN(JSTART+1)=A3-2.3
          XN(JSTART+2)=A3-1.5
          XN(JSTART+3)=A3-1.0
          XN(JSTART+4)=A3-0.5
          XN(JSTART+5)=A3
          XN(JSTART+6)=A3+0.5
          XN(JSTART+7)=A3+1.0
          XN(JSTART+8)=A3+1.55
          JEND=JSTART+9
          XN(JEND)=A3+2.45
      ELSE
          XN(JSTART+1)=-1.35
          XN(JSTART+2)=-0.55
          XN(JSTART+3)=-0.05
          XN(JSTART+4)=0.45
          J=JSTART+5
          IF(X0.GT.2.85)THEN
              XN(J)=1.35
              XN(J+1)=X0-1.49999
              XN(J+2)=X0-0.60
              JEND=J+5
          ELSE
              XN(J+1)=X0-0.60
              XN(J)=0.5*(XN(J+1)+XN(J-1))
              JEND=J+4
          END IF
          XN(JEND-2)=X0-0.10
          XN(JEND-1)=X0+0.45
```

```
              XN(JEND)=XO+1.35
          END IF
          A4=-ALOG(RP)
807       IF(XN(JEND).LT.A4)GO TO 812
          JEND=JEND-1
          GO TO 807
812       JEND=JEND+1
          XN(JEND)=A4
          IA7(I)=JEND
          XPSDM(JEND)=0.0
          JM(I)=JEND
          E4(I)=N-0.5
          E5(I)=NP-N
          XPSDP(JSTART)=0.0
          JP(I)=JSTART
          XPSD(JSTART)=0.0
          DO 900 J=JS,JEND
              A3=EXP(2.0*XN(J))
900           XPSD(J)=E4(I)*ALOG(1.0+A3)+E5(I)*ALOG(1.0+
     1        L2(I)*A3)-XN(J)
C
C  2*SIN(K*X/2)**2 = C1*(K*X)**2  ,  K<=D0
C                  = C2*K**E2     ,  D0<K<=D1
C                  = C3           ,  D1<K
C
C  C1 = COEFFICIENT FOR SMALL (K*X)**2 = 0.5
C  D3 = PEAK VALUE OF APPROXIMATE FUNCTION = 1.855
C  DO = K AT APPROXIMATE FUNCTION PEAK = SQRT(D3/(C1*X**2))
C  D1 = K AT END OF OVERSHOOT = D4/X, D4 = 4.0
C  C2 = COEFFICIENT FOR INTERMEDIATE K = D3/DO**E2
C  C3 = APPROXIMATE FUNCTION AT LARGE K = 1.10
C  E2 = EXPONENT FOR INTERMEDIATE K = ALOG(C2/D3)/ALOG(D1/DO)
C
C         D3=ALOG(1.855)
C         C3=ALOG(1.10)
C         D4=ALOG(4.0)
C         E2=(C3-D3)/(D1-DO)
C         E2=(C3-D3)/(D4-0.5*(D3-C1))
C         C1=ALOG(0.5)
C         C4=0.5*(D3-C1)
```

```
C         E7=E2+1.0
C
      END IF
      CNNXM=0.0
      IF(X.EQ.0.0)RETURN
      XX=ALOG(ABS(X))
      X2=X*X
C     D0=0.5*(D3-XX-XX-C1)
C     D1=D4-XX
C     E2=(C3-D3)/(D1-D0)
C     E7=1.0+E2
      D0=C4-XX
      XN(JSTART)=AMIN1(D0,XN(JSTART+1))-5.6
      XPSD(JSTART)=-XN(JSTART)
      J=IA7(I)
      IF(XN(J).GT.D0)THEN
          D1=D4-XX
          C2=D3-E2*D0
          IF(XN(J).GT.D1)THEN
              JE=J-1
              JT=JE+JSTART
              DO 800 JJ=JSTART,JE
                  J=JT-JJ
800               IF(XN(J).LT.D1)GO TO 850
850           J=J+1
              IF(J.LT.JN(I))THEN
                  JE=JN(I)-1
                  JT=JE+J
                  A4=C3-XPSD(JE+1)
                  F0=EXP(A4)
                  DO 870 JJ=J,JE
                      JS=JT-JJ
                      FN=F0
                      A6=A4
                      A4=C3-XPSD(JS)
      F0 = EXP(A4)
      A0 = A6 - A4
                      IF(ABS(A0).LT.0.0001)FN=(FN-F0)/A0
870                   XPSDN(JS)=XPSDN(JS+1)+FN*(XN(JS+1)-XN(JS))
                  JN(I)=J
```

```
            END IF
            CNNXM=XPSDM(J)
            CT=EXP(D1+D1)
            CT=E4(I)*ALOG(1.0+CT)+E5(I)*ALOG(1.0+L2(I))*CT)
            A6=C3+D1-CT
            FO=EXP(A6)
            A4=C3-XPSD(J)
            A1=EXP(A4)
            A0=A4-A6
            IF(ABS(A0).LT.0.0001)A1=(A1-FO)/A0
            CNNXM=CNNXM+A1*(XN(J)-D1)
        ELSE
            D1=XN(J)
            CT=EXP(D1+D1)
            CT=E4(I)*ALOG(1.0+CT)+E5(I)*ALOG(1.0+L2(I))*CT)
            A6=C2+E7*D1-CT
            FO=EXP(A6)
        END IF
875     J=J-1
        IF(XN(J).LT.DO)GO TO 876
        FN=FO
        A4=C2+E2*XN(J)-XPSD(J)
        FO=EXP(A4)
        A0=A6-A4
        IF(ABS(A0).LT.0.0001)FN=(FN-FO)/A0
        CNNXM=CNNXM+FN*(D1-XN(J))
        D1=XN(J)
        A6=A4
        GO TO 875
876     CT=EXP(DO+DO)
        CT=E4(I)*ALOG(1.0+CT)+E5(I)*ALOG(1.0+L2(I)*CT)
        A4=C2+E7*DO-CT
        A0=A6-A4
        IF(ABS(A0).GT.0.0001)FO=(FO-EXP(A4))/A0
        CNNXM=CNNXM+FO*(D1-DO)
        A6=C1+3.0*DO-CT
        A4=C1+2.0*XN(J)-XPSD(J)
        A1=EXP(A6)
        A0=A6-A4
        IF(ABS(A0).GT.0.0001)A1=(A1-EXP(A4))/A0
```

```
      CNNXM=CNNXM+X2*A1*(DO-XN(J))
   END IF
   IF(J.GT.JP(I)) THEN
      JS=JP(I)
      A6=C1+2.0*XN(JS)-XPSD(JS)
      FN=EXP(A6)
      JS=JS+1
      DO 970 JJ=JS,J
         A4=A6
         FO=FN
         A6=C1+2.0*XN(JJ)-XPSD(JJ)
         FN=EXP(A6)
         AO=A6-A4
         IF(ABS(AO).GT.0.0001) FO = (FN-FO)/AO
970      XPSDP(JJ)=XPSDP(JJ-1)+FO*(XN(JJ)-XN(JJ-1))
      JP(I)=J
   END IF
   CNNXM=CNNXM+X2*XPSDP(J)
   CNNXM=FP(I)*CNNXM
   RETURN
   END
```

APPENDIX D

CALCULATION OF THE FREQUENCY SELECTIVE BANDWIDTH

This appendix establishes the basis and the approximations for the algorithm used to calculate the frequency selective bandwidth, f_0. The transmitted signal is written as

$$s_T(t) = \int_{-\infty}^{\infty} \tilde{S}(f) \exp\{-i2\pi f t\}\, df \tag{D.1}$$

where $\tilde{S}(f)$ is the Fourier transform of the waveform. Since $s_T(t)$ is real,

$$\tilde{S}(f) = \tilde{S}^*(-f) \quad .$$

Also, the signal is assumed narrowband so

$$\begin{aligned} \tilde{S}(f) &= \frac{1}{2}\left[\tilde{P}(f+f_c) + \tilde{P}(f-f_c)\right] \\ \tilde{P}(f) &= \int_{-\infty}^{\infty} p(t) \exp\{-i2\pi f t\}\, dt \end{aligned}$$

where f_c is the carrier frequency and $p(t)$ is the baseband modulation waveform. Also

$$\int_{-\infty}^{\infty} \tilde{S}^*(f)\tilde{S}(f)\, df = 1 \quad .$$

The received waveform is written as

$$s_R(t) = \int_{-\infty}^{\infty} df\, \tilde{S}(f) U(K) \exp\left\{iK \int \sqrt{\epsilon}\, dz - i2\pi f t\right\} \tag{D.2}$$

where $U(K)$ was defined in Appendix B. Non-essential dependences have been suppressed for convenience. Also

$$K = \frac{2\pi f}{c}$$

$$\sqrt{\epsilon} = \sqrt{1 - \frac{f_p^2(z)}{f^2}} \approx 1 - \frac{2\pi e^2\, \overline{n_e(z)}}{mc^2 K^2}$$

$$\overline{n_e(z)} = \text{mean electron density}$$

$$\frac{e^2}{mc^2} = r_0 = \text{classical electron radius } (2.82 \times 10^{-15}\text{m})$$

$$f_p = \text{plasma frequency}$$

After a bit of algebraic manipulation, the signal time of arrival moments can be written as

$$\begin{aligned}
\int_{-\infty}^{\infty} t^n s_R^2(t)dt = \int_0^{\infty} df_2 \Bigg\{ & \tilde{S}(f_2) \exp\left\{iK_2 \int \sqrt{\epsilon_2}\, dz\right\} \left(\frac{i}{2\pi}\right)^n \\
& \cdot \left[\frac{d^n}{df_1^n}\left(G(K_1,K_2)\tilde{S}^*(f_1) \exp\left\{-iK_1 \int \sqrt{\epsilon_1}\, dz\right\}\right)\right]_{f_1=f_2} \\
+ \quad & \tilde{S}^*(f_2) \exp\left\{-iK_2 \int \sqrt{\epsilon_2}\, dz\right\} \left(\frac{-i}{2\pi}\right)^n \\
& \cdot \left[\frac{d^n}{df_1^n}\left(G^*(K_1,K_2)\tilde{S}(f_1) \exp\left\{iK_1 \int \sqrt{\epsilon_1}\, dz\right\}\right)\right]_{f_1=f_2} \Bigg\}
\end{aligned} \tag{D.3}$$

where

$$G(K_1, K_2) = \overline{U^*(K_1)U(K_2)}$$

Since the signal is narrowband, the integrand in Equation D.3 is nonzero and for values of f_2 and f_1 near $\pm f_c$ and K_2 and K_1 near $\pm\frac{2\pi f_c}{c}$. Thus we will develop a solution for $\Gamma(K_1, K_2)$ as an expansion in terms of

$$\eta = \frac{K_2 - K_1}{K_2} = \frac{f_2 - f_1}{f_2}$$

Since

$$\frac{d^n G}{df^n} = \left(-\frac{2\pi}{K_2 c}\right)^n \frac{d^n G}{d\eta^n}$$

we will be able to evaluate terms in Equation D.3. The final result required is the contribution to the standard deviation of the energy time of arrival from the angular scattering of the signal energy. Let

$$\overline{t^n} = \int_{-\infty}^{\infty} t^n s_R^2(t)\, dt$$

The desired result is

$$\sigma_\tau^2 = \left(\overline{t^2} - \overline{t}^2\right)\Big|_{\text{angular scatter}} \tag{D.4}$$

It should be noted that Equation D.3 is a very general result. Various terms account for delay due to the angular scatter, the finite length of the waveform, and the mean total electron content effects.

The equation for the $G(K_1, K_2)$ is

$$\begin{aligned}\frac{\partial G}{\partial z} = &-\frac{i\Delta k}{K_1 K_2}\left(\frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2}\right) - \frac{x}{z}\frac{\partial G}{\partial x} - \frac{y}{z}\frac{\partial G}{\partial y} \\ &-\frac{1}{2}\left[\frac{K_1^2 + K_2^2}{K_1 K_2}\right]\frac{d\sigma_\phi^2}{dz}G + \frac{d\sigma_\phi^2}{dz}\left[1 - D_\phi(\rho(x,y))\right]G\end{aligned} \tag{D.5}$$

where $\Delta k = K_2 - K_1$ and $D_\phi(\rho(x,y))$ is the phase structure function.

Let

$$\begin{aligned}\theta_x &= \frac{x}{z} \\ \theta_y &= \frac{y}{z}\end{aligned}$$

Equation D.5 can now be written

$$\frac{dG}{dz} = -\frac{i}{K_2 z^2}\left(\eta + \eta^2 + \cdots\right)\left(\frac{\partial^2 G}{\partial\theta_x^2} + \frac{\partial^2 G}{\partial\theta_y^2}\right) - \left(1 + \eta + \frac{3}{2}\eta^2 + \cdots\right)\frac{d\sigma_\phi^2}{dz}G$$
$$+ \left(1 + \eta + \eta^2 + \cdots\right)\frac{d\sigma_\phi^2}{dz}\left[1 - D_\phi(\rho(z\theta_x, z\theta_y))\right]G \tag{D.6}$$

where $\frac{d\sigma_\phi^2}{dz}$ is evaluated at $K = K_2$.

Let

$$\begin{aligned}
I(f(z)) &= \int_0^z dz\, f(z) \\
1 - D_\phi(\rho(z\theta_x, z\theta_y)) &= 1 + A_x\theta_x^2 + A_y\theta_y^2 + A_{xy}\theta_x\theta_y \\
&\quad + B_x\theta_x^4 + B_y\theta_y^4 + B_{xy}\theta_x^2\theta_y^2 + \cdots \\
G &= W \exp\left\{-I\left[\left(1 + \eta + \frac{3}{2}\eta^2\right)\frac{d\sigma_\phi^2}{dz}\right]\right\}
\end{aligned}$$

Then

$$\begin{aligned}
\frac{dG}{dz} &= -\frac{i}{K_2 z^2}\left(\eta + \eta^2 + \cdots\right)\left(\frac{\partial^2 W}{\partial\theta_x^2} + \frac{\partial^2 W}{\partial\theta_y^2}\right) \\
&\quad + (1 + \eta + \eta^2 + \cdots)\frac{d\sigma_\phi^2}{dz}\left(1 + A_x\theta_x^2 + \cdots\right)W
\end{aligned} \tag{D.7}$$

Now let

$$W = W_0 + \eta W_1 + \eta^2 W_2$$

Then

$$W_0 = \exp\left\{ I \left[\frac{d\sigma_\phi^2}{dz} \left(1 + A_z\theta_z^2 + \cdots \right) \right] \right\} \tag{D.8}$$

Also

$$\left[\frac{\partial}{\partial z} - \frac{d\sigma_\phi^2}{dz} \left(1 + A_z\theta_z^2 + \cdots \right) \right] W_1 = -\frac{i}{2K_2z^2} \nabla_\perp^2 W_0 + \frac{d\sigma_\phi^2}{dz} \left(1 + A_z\theta_z^2 + \cdots \right) W_0 \tag{D.9}$$

Assume

$$W_1 = W_0 \int_0^z g_1(z')dz'$$

then it is clear that

$$g_1(z)W_0 = -\frac{i}{2K_2z^2} \nabla_\perp^2 W_0 + \frac{d\sigma_\phi^2}{dz} \left(1 + A_z\theta_z^2 + \cdots \right) W_0 \tag{D.10}$$

Also

$$\begin{aligned} \nabla_\perp^2 W_0 = I^2 \left[\frac{d\sigma_\phi^2}{dz} \frac{\partial}{\partial x} \left(1 + A_z\theta_z^2 + \cdots \right) \right] W_0 + I^2 \left[\frac{d\sigma_\phi^2}{dz} \frac{\partial}{\partial y} \left(1 + A_z\theta_z^2 + \cdots \right) \right] W_0 \\ + I \left[\frac{d\sigma_\phi^2}{dz} \nabla_\perp^2 \left(1 + A_z\theta_z^2 + \cdots \right) \right] W_0 \end{aligned} \tag{D.11}$$

Finally,

$$\begin{aligned} g_1(z) &= -\frac{i}{2K_2z^2} I \left[\frac{d\sigma_\phi^2}{dz} (2A_z + 2A_y) \right] && \text{(D.12a)} \\ W_1 &= W_0 I \left\{ -\frac{i}{2K_2z^2} I \left[\frac{d\sigma_\phi^2}{dz} (2A_z + 2B_z) \right] \right\} && \text{(D.12b)} \end{aligned}$$

since in the end we let $\theta_x = \theta_y = 0$.

For W_2

$$\left[\frac{\partial}{\partial z} - \frac{d\sigma_\phi^2}{dz}\left(1 + A_x\theta_x^2 + \cdots\right)\right] W_2 = W_0 g_2(z)$$

$$= -\frac{i}{2K_2z^2}\left(\nabla_\perp^2 W_0 + \nabla_\perp^2 W_1\right)$$

$$+ \frac{d\sigma_\phi^2}{dz}\left(1 + A_x\theta_x^2 + \cdots\right)(W_0 + W_1) \quad \text{(D.13)}$$

where

$$W_2 = W_0 \int_0^z g_2(z')dz'$$

After much algebra

$$g_2(z) = -\frac{i}{2K_2z^2} I\left[\frac{d\sigma_\phi^2}{dz}(2A_x + 2A_y)\right] - \frac{i}{2K_2z^2}\Bigg\{$$

$$I\left[\frac{d\sigma_\phi^2}{dz}(2A_x + 2A_y)\right] I\left[-\frac{i}{2K_2z^2} I\left(\frac{d\sigma_\phi^2}{dz}(2A_x + 2A_y)\right) + \frac{d\sigma_\phi^2}{dz}\right] +$$

$$I\left[-\frac{i}{2K_2z^2}\left[2I^2\left(2A_x\frac{d\sigma_\phi^2}{dz}\right) + 4I^2\left(A_{xy}\frac{d\sigma_\phi^2}{dz}\right) + 2I^2\left(2A_y\frac{d\sigma_\phi^2}{dz}\right)\right.\right.$$

$$\left.\left. + I\left(\frac{d\sigma_\phi^2}{dz}(24B_x + 8B_{xy} + 24B_y)\right)\right] + \frac{d\sigma_\phi^2}{dz}(2A_x + 2A_y)\right] \Bigg\}$$

$$+\frac{d\sigma_\phi^2}{dz} I\left[-\frac{i}{2K_2z^2} I\left(\frac{d\sigma_\phi^2}{dz}(2A_x + 2A_y)\right) + \frac{d\sigma_\phi^2}{dz}\right] \quad \text{(D.14)}$$

Finally

$$G = \exp\left\{-I\left[\left(\eta + \frac{3}{2}\eta^2 + \cdots\right)\frac{d\sigma_\phi^2}{dz}\right]\right\}\left[1 + \eta I(g_1) + \eta^2 I(g_2) + \cdots\right]$$

After expanding the exponential

$$G = 1 + \eta G_1 + \eta^2 G_2 + \cdots \tag{D.15}$$

$$G_1 = -i\, I\left\{\frac{1}{2K_2 z^2} I\left[\frac{d\sigma_\phi^2}{dz}(2A_x + 2A_y)\right]\right\} \tag{D.16}$$

$$\begin{aligned}
G_2(z) = &-\frac{\sigma_\phi^2}{2} - I\left\{\frac{1}{2K_2 z^2} I\left[\frac{d\sigma_\phi^2}{dz}(2A_x + 2A_y)\right] I\left[\frac{1}{2K_2 z^2} I\left(\frac{d\sigma_\phi^2}{dz}(2A_x + 2A_y)\right)\right]\right.\\
&+\frac{1}{2K_2 z^2} I\left[\frac{1}{2K_2 z^2}\left(2I^2\left(2\frac{d\sigma_\phi^2}{dz}A_x\right) + 4I^3\left(\frac{d\sigma_\phi^2}{dz}A_{xy}\right)\right.\right.\\
&\left.\left.\left.+2I^2\left(2\frac{d\sigma_\phi^2}{dz}A_y\right) + I\left(\frac{d\sigma_\phi^2}{dz}(24B_x + 8B_{xy} + 24B_y)\right)\right)\right]\right\}\\
&+i\left\{I\left[-\frac{1}{2K_2 z^2} I\left(\frac{d\sigma_\phi^2}{dz}(2A_x + 2A_y)\right)\right.\right.\\
&\quad -\frac{1}{2K_2 z^2} I\left(\frac{d\sigma_\phi^2}{dz}(2A_x + 2A_y)\right)\left(1 + I\left(\frac{d\sigma_\phi^2}{dz}\right)\right)\\
&\quad \left.-\frac{d\sigma_\phi^2}{dz} I\left(\frac{1}{2K_2 z^2} I\left(\frac{d\sigma_\phi^2}{dz}(2A_x + 2A_y)\right)\right)\right]\\
&\quad \left.+I\left(\frac{d\sigma_\phi^2}{dz}\right) I\left(\frac{1}{2K_2 z^2} I\left(\frac{d\sigma_\phi^2}{dz}(2A_x + 2A_y)\right)\right)\right\}
\end{aligned} \tag{D.17}$$

Moments in Equation D.3 can now be evaluated. Let us first calculate the contribution to the first moment from the mean plasma density. This requires the calculation of

$$\frac{d}{df}\left(\exp\left\{-iK\int_0^{z_t}\sqrt{\epsilon}\,dz\right\}\right) = -i\left[\frac{2\pi z_t}{c} + \frac{c\tau_0}{f^2}\int_0^{z_t}\overline{n_e(z)}dz\right]\exp\left\{-iK\int_0^{z_t}\sqrt{\epsilon}\,dz\right\}$$

Thus

$$\int_{-\infty}^{\infty} ts_R^2(t)dt|_{\overline{n_e}} = \int_{-\infty}^{\infty} df_2 \tilde{S}^*(f_2)\tilde{S}(f_2)\left(\frac{z_t}{c} + \frac{cr_0}{2\pi f^2}\int_0^{z_t} \overline{n_e(z)}dz\right) \quad \text{(D.18a)}$$

$$= \frac{z_t}{c} + \frac{cr_0}{2\pi f^2}\int_0^{z_t} \overline{n_e(z)}dz \quad \text{(D.18b)}$$

The first term is the signal propagation time at the speed of light and the second term is the well known group delay.

Now let us look at the contribution to the mean delay from the scintillation term

$$\frac{\partial G}{\partial f_1} = -\frac{2\pi}{K_2 c}(G_1 + 2\eta G_2 + \cdots)\,|_{\eta=0} \quad \text{(D.19a)}$$

$$= i\frac{2\pi c}{(2\pi f_c)^2} I\left\{\frac{1}{z^2} I\left[\frac{d\sigma_\phi^2}{dz}(A_x + A_y)\right]\right\} \quad \text{(D.19b)}$$

$$\int_{-\infty}^{\infty} ts_R^2(t)dt|_G = -\frac{c}{(2\pi f_2)^2} I\left\{\frac{1}{z^2} I\left[\frac{d\sigma_\phi^2}{dz}(A_x + A_y)\right]\right\} \quad \text{(D.20)}$$

After a horrible amount of algebra (the proverbial exercise for the reader), the variance of the delay from the scatter is

$$\sigma_t^2|_G = \frac{\sigma_\phi^2}{(2\pi f_2)^2} + \frac{c^2}{2(2\pi f_2)^2} I\Bigg\{ \frac{1}{z^2} I\Bigg[\frac{1}{z^2}$$

$$\cdot \Bigg[I\left(\frac{d\sigma_\phi^2}{dz}(24B_x + 8B_{xy} + 24B_y)\right) + 2I^2\left(\frac{d\sigma_\phi^2}{dz}2A_x\right)$$

$$+2I^2\left(\frac{d\sigma_\phi^2}{dz}2A_y\right) + 4I^2\left(\frac{d\sigma_\phi^2}{dz}A_{xy}\right)\Bigg]\Bigg] \Bigg\} \quad \text{(D.21)}$$

The first term is from the group delay jitter and we drop it since creates no waveform distortion. Group delay jitter is generally handled separately. Now let us look at the B terms. For the often used "K^{-2}" spectrum

$$24B_x \approx \frac{1}{2\ell^2 L_x^2}$$

$$A_x \approx -\frac{\ln(L_x/\bar{x})}{2L_x^2}$$

The ratio of the contribution from these terms is approximately

$$\frac{L_x^2}{\sigma_\phi^2 \ell_x^2 \ln^2(L_x/x)} \ll 1$$

where

$$L_x = \text{outer scale}$$

$$\ell_x = \text{inner scale}$$

This term is always small for satellite frequencies and applications. Thus it is dropped for efficiency. The B term also lead to pecular results. As ℓ_x goes to zero, the delay becomes infinite. The signal energy, however, is still reasonably described by the A terms. The divergence is traceable to a common problem of moment methods with functions that do not go to zero rapidly for large arguments. Sufficiently high moments will diverge while the function is still integrable. Thus dropping the B terms provides insurance against unreasonable results.

For the power spectrums described earlier in this report the A coefficients are related to the electron density scale sizes.

$$A_x = -B_2 \frac{L_x^2 z^2}{L_x^2 L_y^2 - L_{xy}^2}$$

$$A_y = -B_2 \frac{L_y^2 z^2}{L_x^2 L_y^2 - L_{xy}^2}$$

$$A_{xy} = -B_2 \frac{2L_{xy} z^2}{L_x^2 L_y^2 - L_{xy}^2}$$

The derivation of B_2 is described earlier leading up to Equation 1.21. This particular method insures reasonable results even if the second moment in delay is not well defined. Finally we set

$$
\begin{aligned}
f_2'^2 &= \frac{1}{4\pi^2 \sigma_\tau^2|_G} \\
f_0 &= f_2'(1 + C_1^2)^{-1/2}
\end{aligned}
$$

where $0 \leq C_1 \leq 0.25$. The C_1 parameter was added to ease the numerical handling of the generalized power spectrum in the CIRF computer program which provided explicit representations of scintillated signals. Currently it is chosen in ACIRF, the successor to CIRF, depending on the specific channel variation being run.

The integral in Equation D.21 can be simplified. The original form can be expressed as

$$
\int_0^{z_t} \frac{dx}{x^2} \int_0^{x} \frac{dy}{y^2} \left[\int_0^{y} dz\, z^2 f(z) \right]^2 = \int_0^{z_t} dx (z_t - x)^2 f(x) \int_0^{x} dz\, z^2 f(z)
$$

The latter form is much easier to implement and is strongly recommended.

APPENDIX E

AN ALGORITHM FOR CALCULATING THE SCINTILLATION AND ABSORPTION OF RADIO SIGNALS

This appendix lists SUBROUTINE PROP which calculates the scintillation and absorption of radio signals. The inputs and outputs are documented in the comments section at the beginning of the subroutine. This routine implements the algorithm described in the body of this paper for an environment consisting of a series of slabs each with its particular properties. Thus the subroutine can handle most situations.

```
      SUBROUTINE PROP(MODE)
C
C  PROP, VERSION 6, 10 FEB 90
C
C  THIS ROUTINE CALCULATES PARAMETERS DESCRIBING THE PROPERTIES
C  OF RADIO SIGNALS THAT HAVE PROPAGATED THROUGH AN ABSORBING
C  AND STRUCTURED MEDIUM.  THE ABSORPTION MODEL INCLUDES
C  ELECTRON-ION AND ELECTRON-NEUTRAL COLLISIONS.  THE PARAMETERS
C  DESCRIBING THE SCINTILLATION OF RADIO SIGNALS ASSUMES
C  PROPAGATION THROUGH A STRUCTURED PLASMA DESCRIBED BY THE
C  WITTWER/KILB STRUCTURE MODEL.  THE LIMITS ON INPUT PARAMETERS
C  ARE DESCRIBED BELOW.  FOR EFFICIENCY, NO CHECKS ARE MADE TO
C  TEST THE INPUTS FOR LEGALITY EXCEPT FOR THE NUMBER OF
C  ENVIRONMENT LAYERS.
C
C  THIS PROGRAM ASSUMES THAT ANY INPUTS HAVE AT LEAST A REMOTE
C  RESEMBLENCE TO PHYSICAL REALITY.  SINCE I CAN NOT PROTECT
C  AGAINST ALL CONCEIVABLE POSSIBILITIES, I ONLY CLAIM THAT THE
C  REASONABLENESS OF THE OUTPUTS IS AT LEAST COMPARABLE TO THE
C  REASONABLENESS OF THE INPUTS.
C
C  THIS SUBROUTINE WAS PROGRAMMED BY DR. LEON A. WITTWER.
C  QUESTIONS MAY BE REFERRED TO DR. WITTWER AT 703-325-7028.
C
C  ERRORS OR WARNINGS ARE OUTPUT VIA WRITE(*,....) CALLS.  "*"
```

```
C  IS ASSUMED DEFAULTED TO THE TERMINAL.
C
C  MODE CONTROLS THE PROPERTIES CALCULATED AND RETURNED.  SEE
C  BELOW FOR DETAILS.
C
C  LABELED COMMON /PRINPT/ CONTAINS INPUT TO THE SUBROUTINE
C
C    F = CARRIER FREQUENCY(HZ).
C    NLYRS = NUMBER OF ENVIRONMENT LAYERS.
C            (NLYRS.LE.NIP WHERE NIP = MAXIMUM NUMBER OF LAYERS
C            ALONG THE LINE OF SIGHT.  NIP IS SET IN A PARAMETER
C            STATEMENT BELOW.)
C    LOSV(I) = VECTOR PARALLEL TO LINE OF SIGHT.  THE USER
C              COORDINATE SYSTEM MUST BE A RIGHT HANDED
C              CARTESIAN SYSTEM.  THIS VECTOR DEFINES THE LOCAL
C              Z COORDINATE ALONG THE LINE OF SIGHT.
C    LOS(I) = LINE OF SIGHT COORDINATES(KM) OF ENVIRONMENT
C             LAYER CENTERS.  ONE END OF THE LINK IS AT Z=0
C             AND THE OTHER END IS AT Z= LOS(NLYRS+1).  ANTENNA
C             RELATED OUTPUTS (CPT, CQT, AND TWPCC) ASSUME THAT
C             THE ANTENNA IS AT Z=LOS(NLRYS+1).
C             (LOS(I+1).GT.LOS(I) FOR ALL I)
C    DLOS(I) = LAYER THICKNESS(KM) OF LAYER CENTERED AT LOS(I).
C    NE(I) = MEAN ELECTRON DENSITY(/CM**3) IN LAYER CENTERED
C            AT LOS(I).
C    NE2(I) = ELECTRON DENSITY VARIANCE(/CM**6) IN LAYER
C             CENTERED AT LOS(I).
C    ND(I) = NEUTRAL DENSITY(/CM**3) IN LAYER CENTERED AT LOS(I).
C    TE(I) = MEAN PLASMA TEMPERATURE(DEG KELVIN) IN LAYER CENTERED
C            AT LOS(I).
C    BV(J,I) = VECTOR PARALLEL TO THE EARTH'S MAGNETIC FIELD
C              AT LOS(I).  THE MODEL ALLOWS FOR NONISOTROPIC
C              STRUCTURE IN ONE DIRECTION DESIGNATED BY BV(J,I).
C              J IS THE CARTESIAN COORDINATE INDEX.  STRUCTURE
C              IS ASSUMED ISOTROPIC ABOUT THE VECTOR BV.
C    LO(I) = OUTER SCALE(KM) PERPENDICULAR TO BV AT LOS(I).
C    LF(I) = FREEZING SCALE(KM) PERPENDICULAR TO BV AT LOS(I).
C            (LF(I).LE.LO(I).AND.LF(I).GE.1.0E-7*LO(I))
C    LI(I) = INNER SCALE(KM) PERPENDICULAR TO BV AT LOS(I).
C            (LI(I).LT.LF(I))
```

```
C    LT(I) = OUTER SCALE(KM) PARALLEL TO BV AT LOS(I).  THE
C            FREEZING SCALE PARALLEL TO BV VECTOR IS LT(I)*
C            LF(I)/LO(I).  THE INNER SCALE PARALLEL TO BV IS
C            LT(I)*LI(I)/LO(I).
C            (LT(I).LE.100.0*LO(I))
C    N(I)   2.0*N(I)-2.0 IS INTERMEDIATE SCALE SPECTRAL INDEX.
C           (N(I).GE.1.6.AND.N(I).LE.2.0)
C    NP(I)   2.0*NP(I)-2.0 IS SMALL SCALE SPECTRAL INDEX.
C            (NP(I).GE.2.0.AND.NP(I).LE.4.0)
C    VST(J,I) = VECTOR STRUCTURE VELOCITY(KM/SEC) AT LOS(I).
C               J IS THE CARTESIAN COORDINATE INDEX.
C    VTR(J) = TRANSMITTER VELOCITY(KM/SEC). J IS THE CARTESIAN
C             COORDINATE INDEX.
C    VRE(J) = RECEIVER VELOCITY(KM/SEC). J IS THE CARTESIAN
C             COORDINATE INDEX.
C
C  THE OUTPUT IS DETERMINED BY THE VALUE OF MODE.
C
C    MODE = 0
C      LABELED COMMON /PROUTO/
C        TO = SIGNAL DECORRELATION TIME(SEC)
C        TWPCC = TWO WAY PATH SIGNAL CORRELATION COEFFICIENT
C                ASSUMING THAT THE TRANSMITTER/RECEIVER IS AT
C                Z=LOS(NLYRS+1).
C        CPT = TIME SPACE CROSS CORRELATION COEFFICIENT IN P
C              DIRECTION AT Z=LOS(NLYRS+1)
C        CQT = TIME SPACE CROSS CORRELATION COEFFICIENT IN Q
C              DIRECTION AT Z=LOS(NLYRS+1)
C        FO = FREQUENCY SELECTIVE BANDWIDTH(HZ)
C        X2 = MEAN SQUARE LOG AMPLITUDE FLUCTUATION
C        KA = ABSORPTION(DB)
C        AT = ANTENNA TEMPERATURE(DEG KELVIN) AT RECEIVER.
C             ASSUMES FIREBALL FILLS ENTIRE ANTENNA.  THE
C             RETURNED TEMPERATURE IS FOR THE PLASMA BETWEEN
C             THE TRANSMITTER AND RECEIVER.  IT DOES NOT
C             INCLUDE ANY PLASMA BEHIND THE TRANSMITTER.
C        LPV = VECTOR IN DIRECTION OF MINIMUM DECORRELATION
C              DISTANCE AT Z=LOS(NLYRS+1) DEFINED AS THE P
C              DIRECTION.
C        LP = MINIMUM DECORRELATION DISTANCE AT Z=LOS(NLYRS+1)(M).
```

```
C        LQ = MAXIMUM DECORRELATION DISTANCE AT Z=LOS(NLYRS+1)(M).
C             THE DIRECTION IS PERPENDICULAR TO THE VECTOR LPV
C             AND THE LOS.
C        LPPV = VECTOR IN DIRECTION OF MINIMUM DECORRELATION
C               DISTANCE AT Z=0.
C        LPP = MINIMUM DECORRELATION DISTANCE AT Z=0(M).
C        LQP = MAXIMUM DECORRELATION DISTANCE AT Z=0(M).
C              THE DIRECTION IS PERPENDICULAR TO THE VECTOR LPPV
C              AND THE LOS.
C        CPPP = TRANSMITTER-RECEIVER CORRELATION COEFFICIENT IN
C               DIRECTION OF MINIMUM DECORRELATION DISTANCE AT
C               Z=0.
C    MODE = 1
C      ALL OF THE ABOVE OUTPUTS AND
C      LABELED COMMON /PROUT1/
C        P2 = MEAN SQUARE PHASE FLUCTUATION(RAD**2)
C        TEC = TOTAL ELECTRON CONTENT(/CM**2)
C        LOEF = EFFECTIVE OUTER SCALE(KM)
C        LFEF = EFFECTIVE FREEZING SCALE(KM)
C        LIEF = EFFECTIVE INNER SCALE(KM)
C        NEF   2.0*NEF-2.0 IS EFFECTIVE INTERMEDIATE SCALE
C              SPECTRAL INDEX
C        NPEF   2.0*NPEF-2.0 IS EFFECTIVE SMALL SCALE SPECTRAL
C               INDEX
C        VEF = EFFECTIVE TEC VELOCITY
C              IF VEF IS ZERO THEN LOEF, LFEF, LIEF, NEF, NPEF,
C              KR, FR, DLM(J), AND DPM(J) ARE UNDEFINED.
C    MODE = 2
C      ALL OF THE ABOVE OUTPUTS AND
C      LABELED COMMON /PROUT2/
C        KR = RAYLEIGH WAVENUMBER(1.0/KM)
C        FR = RAYLEIGH FREQUENCY(HZ)
C        DL = MEAN GROUP DELAY(SEC)
C        DLM(1) = 3 SIGMA DELAY(SEC)
C        DLM(2) = 3 SIGMA DELAY RATE(SEC/SEC)
C        DLM(3) = 3 SIGMA DELAY ACCELERATION(SEC/SEC**2)
C        DLM(4) = 3 SIGMA DELAY JERK(SEC/SEC**3)
C        DP = MEAN PHASE(RAD)
C        DPM(1) = 3 SIGMA PHASE(RAD)
C        DPM(2) = 3 SIGMA DOPPLER(HZ)
```

```
C        DPM(3) = 3 SIGMA DOPPLER RATE(HZ/SEC)
C        DPM(4) = 3 SIGMA JERK(HZ/SEC**2)
C
C  SET PARAMETERS
C
C    NIP = MAXIMUM NUMBER OF ENVIRONMENT LAYERS.  NIP MUST ALSO
C          BE SET IN FUNCTIONS CNNXMI AND CNNXM.
C    RC = RAYLEIGH CRITERIA
C    PND = DEFAULT PHASE NOISE
C    C1 = FREQUENCY SELECTIVE PARAMETER
C    XLEST = INITIAL SCALE ESTIMATE
C    PI02 = PI/2
C    PI04 = PI/4
C    TMPK = METERS/KILOMETER = 1000.0
C
      PARAMETER (NIP=11,RC=0.1,PND=0.025,C1=0.25,XLEST=1.0E-5)
      PARAMETER (TMPK=1000.0,PI02=1.5707963,PI04=0.78539816)
C
      REAL LOS,NE,LO,LF,LI,LT,N,NP,NE2,LOSV,ND
      REAL LP,LPV,LQ,KA
      REAL LOEF,LFEF,NEF,NPEF,LIEF
      REAL LPP,LPPV,LQP
      REAL KR
      REAL LU,LV,LY2,LUP,LVP
      REAL LOSVA
      DIMENSION LOS(NIP+1),DLOS(NIP),NE(NIP),TE(NIP),BV(3,NIP)
      DIMENSION LF(NIP),LI(NIP),LT(NIP),N(NIP),NP(NIP),NE2(NIP)
      DIMENSION VST(3,NIP),LO(NIP),LOSV(3),ND(NIP),VTR(3),VRE(3)
      DIMENSION LPV(3)
      DIMENSION LPPV(3)
      DIMENSION DLM(4),DPM(4)
      DIMENSION LY2(NIP),DP2(NIP),VT(NIP),UT(NIP),UVT(NIP)
      DIMENSION X(3),ARG(NIP),U(3),V(3),Q2(NIP),XN(20)
      DIMENSION UV(NIP),VV(NIP)
      COMMON /PRINPT/F,NLYRS,LOSV,LOS,DLOS,NE,NE2,ND,TE,BV,LO,LF
     1 ,LI,LT,N,NP,VST,VTR,VRE
      COMMON /PROUTO/TO,TWPCC,FO,X2,KA,AT,LPV,LP,LQ,CPT,CQT,
     1    LPP,LPPV,LQP,CPPP
      COMMON /PROUT1/P2,TEC,LOEF,LFEF,LIEF,NEF,NPEF,VEF
      COMMON /PROUT2/KR,FR,DL,DLM,DP,DPM
```

```
      COMMON /CNDATA/QN,QNP,R,RP
      COMMON /CNNCOM/CLIM,ARG,DP2
C
C  INITIAL SET UP FOR PROPAGATION CALCULATION AND CALCULATE
C  SIMPLE INTEGRAL PROPERTIES(P2,KA,TEC,AT).
C
      IF(NLYRS.GT.NIP)THEN
          WRITE(*,6)
6         FORMAT(1X,'FATAL ERROR--TO MANY LOS INTEGRATION',
     1      ' POINTS')
          PAUSE 'CR TO END'
          STOP
      END IF
C
C  L = INDEX OF LARGEST LOSV COMPONENT
C
      TP1=ABS(LOSV(1))
      L=1
      DO 30 J=2,3
      IF(ABS(LOSV(J)).LT.TP1)GO TO 30
      L=J
      TP1=ABS(LOSV(J))
30    CONTINUE
      ZT=LOS(NLYRS+1)
      LOSVA=SQRT(LOSV(1)*LOSV(1)+LOSV(2)*LOSV(2)+LOSV(3)*LOSV(3))
      P2=0.0
      KA=0.0
      AT=0.0
      TEC=0.0
      VZ=0.0
      X2=0.0
      DO 1000 I=1,NLYRS
      TP3=BV(1,I)*BV(1,I)+BV(2,I)*BV(2,I)+BV(3,I)*BV(3,I)
C
C  CALCULATE LOCAL X DIRECTION VECTOR
C
      X(1)=BV(2,I)*LOSV(3)-BV(3,I)*LOSV(2)
      X(2)=BV(3,I)*LOSV(1)-BV(1,I)*LOSV(3)
      X(3)=BV(1,I)*LOSV(2)-BV(2,I)*LOSV(1)
      TP1=X(1)*X(1)+X(2)*X(2)+X(3)*X(3)
```

```
      IF(TP1/(TP3*LOSVA*LOSVA).LT.1.0E-08)THEN
          LY2(I)=LO(I)*LO(I)
          J=L+1
          IF(J.GT.3)J=1
          X(J)=1.0
          X(L)=-LOSV(J)/LOSV(L)
          J=L-1
          IF(J.EQ.0)J=3
          X(J)=0.0
          TP1=1.0+X(L)*X(L)
      ELSE
C
C  CALCULATE LOCAL SQUARE OF OUTER SCALE, LY2, IN LOCAL Y
C  DIRECTION.  THE LOCAL SQUARE OF OUTER SCALE IN X DIRECTION
C  IS LO*LO.
C
          TP4=(BV(1,I)*LOSV(1)+BV(2,I)*LOSV(2)+BV(3,I)*LOSV(3))
          LY2(I)=((LO(I)*TP4)**2+LT(I)*LT(I)*TP1)/(LOSVA*LOSVA
     1      *TP3)
      END IF
      TP2=SQRT(TP1)
      IF(I.EQ.1)THEN
C
C  SET INTERNAL REFERENCE VECTORS, U AND V
C
          U(1)=X(1)/TP2
          U(2)=X(2)/TP2
          U(3)=X(3)/TP2
          V(1)=(LOSV(2)*U(3)-LOSV(3)*U(2))/LOSVA
          V(2)=(LOSV(3)*U(1)-LOSV(1)*U(3))/LOSVA
          V(3)=(LOSV(1)*U(2)-LOSV(2)*U(1))/LOSVA
      END IF
C
C  SET UP LOCAL TRANSFORMATION MATRICES FOR U,V COORDINATES
C
      J=L+1
      IF(J.GT.3)J=1
      M=J+1
      IF(M.GT.3)M=1
      TP1=(X(1)*U(1)+X(2)*U(2)+X(3)*U(3))/TP2
```

```
      TP2=LOSVA*(X(J)*U(M)-X(M)*U(J))/(TP2*LOSV(L))
      TP3=LO(I)*LO(I)
      UT(I)=TP1*TP1/TP3+TP2*TP2/LY2(I)
      VT(I)=TP1*TP1/LY2(I)+TP2*TP2/TP3
      UVT(I)=2.0*TP1*TP2*(1.0/TP3-1.0/LY2(I))
C
C  CALCULATE LOCAL VELOCITIES IN U AND V DIRECTIONS
C
      DO 901 J=1,3
901   X(J)=VST(J,I)-(LOS(I)*VRE(J)+(ZT-LOS(I))*VTR(J))/ZT
      UV(I)=U(1)*X(1)+U(2)*X(2)+U(3)*X(3)
      VV(I)=V(1)*X(1)+V(2)*X(2)+V(3)*X(3)
C
C  INITIALIZE CNNXM FOR ITH LAYER
C
C    QN = n
C    QNP = n'
C    R = FREEZING SCALE / OUTER SCALE
C    RP = INNER SCALE / OUTER SCALE
C
      QN=N(I)
      QNP=NP(I)
      R=LF(I)/LO(I)
      RP=LI(I)/LO(I)
      TP2=CNNXM(RP,-I)
C
C  TP1=N2(n,n',R)
C
      TP1=1.0+((0.06*QNP-0.54)*QNP+0.38)*ALOG(R)
      Q2(I)=1.0/(1.0-(QNP-QN)*(TP1*R)**(QN+QN-2.0)/(QNP-1.0))
C
C  TP2=N3(n,n',R)
C
      TP2=(0.12*QNP-1.3)*QNP+2.18+((-0.1*QNP+1.11)*QNP-0.49)
     1 /SQRT(R)
      TP2=TP2/SQRT(1.0+0.3/R)
      TP2=1.0/(1.0-(QNP-QN)*(TP2*R)**(QN+QN-3.0)/(QNP-1.5))
C
C  TP3 = EFFECTIVE PARALLEL SCALE SIZE.
C
```

```
      TP3=LO(I)*LT(I)/SQRT(LY2(I))
C
C THE FOLLOWING IS USED TO ELIMINATE EXPLICIT CALCULATION OF
C THE GAMMA FUNCTION
C     (0.12+0.77/N)=GAMMA(N-0.5)/(SQRT(PI)*GAMMA(N))
C
C  TP1 = INTEGRATED VARIANCE OF PROCESS DESCRIBED BY NE2
C
      TP1=2.0*(QN-1.5)*TP2*TP3*DLOS(I)/(Q2(I)*(QN-1.0)*(0.12
     1 +0.77/QN))
C
C  DP2(I) = INTEGRATED PHASE VARIANCE FOR ITH LAYER
C
      DP2(I)=7.12E+05*TP1*NE2(I)/(F*F)
      P2=P2+DP2(I)
C
C  GET DOMINANT LAYER, TRY TO SAVE SOME TIME
C
      TP1=DP2(I)*(UT(I)+VT(I))
      IF(TP1.GT.X2)THEN
          IDL=I
          X2=TP1
      END IF
C
C  AT = ANTENNA TEMPERATURE(DEG KELVIN)
C  KA = ABSORPTION(DB)
C
C  TP15 = ELECTRON ION ABSORPTION(DB/KM)
C
      TP1=SQRT(NE2(I)+NE(I)*NE(I))
      TP2=1.8*TP1*ALOG(1.3E16*TE(I)**3/(F*F))/TE(I)**1.5
      TP15=4.6E4*TP1*TP2/(39.44*F*F+TP2*TP2)
C
C  TP14 = ELECTRON NEUTRAL ABSORPTION(DB/KM)
C
      TP3=2.0E-11*ND(I)*TE(I)
      TP4=(157.0*F/TP3)**0.75
      TP4=2.0+(1.0-TP4)/(1.0+TP4)
      TP16=TP3/(8.8*F)
      TP3=TP3*(0.8+0.2*(1.0-TP16)/(1.0+TP16))
```

```
      TP14=4.6E4*NE(I)*TP3/((6.3*F*TP4)**2+TP3*TP3)
      TP2=TP15+TP14
      TP1=EXP(-0.23*TP2*DLOS(I))
      AT=TE(I)*(1.0-TP1)+AT*TP1
      KA=KA+TP2*DLOS(I)
C
C  TEC = TOTAL ELECTRON CONTENT(/CM**2)
C
      TEC=TEC+1.0E5*NE(I)*DLOS(I)
C
C  ACCUMULATE FOR EFFECTIVE PLASMA LOS VELOCITY
C
      VZ=VZ+(VST(1,I)*LOSV(1)+VST(2,I)*LOSV(2)+VST(3,I)
     1  *LOSV(3))*DP2(I)
1000  CONTINUE
C
C  VZ = EFFECTIVE PLASMA VELOCITY PARALLEL TO LOS
C
      IF(P2.GT.0.0)THEN
          VZ=VZ/(LOSVA*P2)
      ELSE
          VZ=0.0
      END IF


C
C  BEGIN CALCULATION OF PROPERTIES THAT REQUIRE ITERATION OVER
C  LOS INTEGRALS OF THE STRUCTURE FUNCTION AND OTHER COMPLICATED
C  THINGS
C
C  CALCULATE LU, LV, AND CUV
C
      DO 1100 I=1,NLYRS
1100  ARG(I)=SQRT(UT(I))*LOS(I)/ZT
      LU=CNNXNI(NLYRS,XLEST)
      DO 1200 I=1,NLYRS
1200  ARG(I)=SQRT(VT(I))*LOS(I)/ZT
      LV=CNNXNI(NLYRS,XLEST)
      TP6=1.0
      IF(UVT(IDL).LT.0.0)TP6=-TP6
```

```
      X2 = 0.56
1325  DO 1300 I=1,NLYRS
1300  ARG(I)=SQRT(LU*LU*UT(I)+LV*LV*VT(I)-TP6*LU*LV*UVT(I))*
     1 LOS(I)/ZT
      CUV=CNNXMI(NLYRS,0.1)
      IF(CUV.GT.X2)GO TO 1320
      TP6=-TP6
      X2 = 0.0
      GO TO 1325
1320  CUV=TP6*(1.0-CLIM/(2.0*CUV*CUV))
C
C  CALCULATE LP, LPV, AND LQ
C
      TP4=LU/LV-LV/LU
      IF(ABS(TP4).GT.1.0E-7)THEN
          TP2=ATAN(2.0*CUV/TP4)/2.0
      ELSE
          TP2=PI04
      ENDIF
      TP20=SIN(TP2)
      TP21=COS(TP2)
      TP1=TP20/LV
      TP2=TP21/LU
      TP22 = TP1*TP1+TP2*TP2-2.0*CUV*TP1*TP2
      TP1=TP20/LU
      TP2=TP21/LV
      TP23 = TP1*TP1+TP2*TP2+2.0*CUV*TP1*TP2
      IF(TP22 .GT. TP23)THEN
          IF(TP23.LT.1.0E-04*TP22)TP23=1.0E-04*TP22
          LP=1.0/SQRT(TP22)
          LQ=1.0/SQRT(TP23)
      ELSE
          IF(TP22.LT.1.0E-04*TP23)TP22=1.0E-04*TP23
          LP=1.0/SQRT(TP23)
          LQ=1.0/SQRT(TP22)
          TP1=TP20
          TP20=TP21
          TP21=-TP1
      ENDIF
      DO 1515 J=1,3
```

```
1515  LPV(J)=U(J)*TP21+V(J)*TP20
C
C  CALCULATE LUP, LVP, AND CUVP
C
      DO 3100 I=1,NLYRS
3100  ARG(I)=SQRT(UT(I))*(ZT-LOS(I))/ZT
      LUP=CNNXMI(NLYRS,XLEST)
      DO 3200 I=1,NLYRS
3200  ARG(I)=SQRT(VT(I))*(ZT-LOS(I))/ZT
      LVP=CNNXMI(NLYRS,XLEST)
      X2=0.56
3325  DO 3300 I=1,NLYRS
3300  ARG(I)=SQRT(LUP*LUP*UT(I)+LVP*LVP*VT(I)-TP6*LUP*LVP*UVT(I)
     1 )*(ZT-LOS(I))/ZT
      CUVP=CNNXMI(NLYRS,0.1)
      IF(CUVP.GT.X2)GO TO 3320
      TP6=-TP6
      X2=0.0
      GO TO 3325
3320  CUVP=TP6*(1.0-CLIM/(2.0*CUVP*CUVP))
C
C  CALCULATE LPP, LPPV, AND LQP
C
      TP4=LUP/LVP-LVP/LUP
      IF(ABS(TP4).GT.1.0E-7)THEN
          TP2=ATAN(2.0*CUVP/TP4)/2.0
      ELSE
          TP2=PI04
      END IF
      TP3=SIN(TP2)
      TP4=COS(TP2)
      TP1=TP3/LVP
      TP2=TP4/LUP
      TP22 = TP1*TP1+TP2*TP2-2.0*CUVP*TP1*TP2
      TP1=TP3/LUP
      TP2=TP4/LVP
      TP23 = TP1*TP1+TP2*TP2+2.0*CUVP*TP1*TP2
      IF(TP22.GT.TP23)THEN
          IF(TP23.LT.1.0E-04*TP22)TP23=1.0E-04*TP22
          LPP=1.0/SQRT(TP22)
```

```
         LQP=1.0/SQRT(TP23)
      ELSE
         IF(TP22.LT.1.0E-04*TP23)TP22=1.0E-04*TP23
         LPP=1.0/SQRT(TP23)
         LQP=1.0/SQRT(TP22)
         TP1=TP3
         TP3=TP4
         TP4=-TP1
      ENDIF
      DO 3515 J=1,3
3515  LPPV(J)=U(J)*TP4+V(J)*TP3
C
C  CALCULATE CPPP
C
      TP1=TP20/LVP
      TP2=TP21/LUP
      TP3=1.0/SQRT(TP1*TP1+TP2*TP2-2.0*CUVP*TP1*TP2)
      DO 3700 I=1,NLYRS
      TP4=ABS(LP*LOS(I)-TP3*(ZT-LOS(I)))
3700  ARG(I)=TP4*SQRT(TP21*TP21*UT(I)+TP20*TP20*VT(I)
     1 -TP20*TP21*UVT(I))/ZT
      TP1=CNNXMI(NLYRS,0.1)
      CPPP=-1.0+CLIM/(2.0*TP1*TP1)
      IF(CPPP.LT.-0.9999)CPPP=-0.9999
C
C  CALCULATE PERPENDICULAR TO
C
      TOPRP=1.0E-06
      DO 2000 I=1,NLYRS
2000  ARG(I)=SQRT(UV(I)*UV(I)*UT(I)+VV(I)*VV(I)*VT(I)-UV(I)*
     1 VV(I)*UVT(I))
      TOPRP=CNNXMI(NLYRS,TOPRP)
C
C  CALCULATE TWO WAY PATH CORRELATION COEFFICIENT
C
      TWPCC=0.0
      DO 2002 I=1,NLYRS
      DO 2001 J=1,3
2001  X(J)=VST(J,I)-LOS(I)*VRE(J)/ZT
      TP1=X(1)*U(1)+X(2)*U(2)+X(3)*U(3)
```

```
      TP2=X(1)*V(1)+X(2)*V(2)+X(3)*V(3)
      TP3=6.67E-06*LOS(I)*SQRT(TP1*TP1*UT(I)+TP2*TP2*
     1  VT(I)-TP1*TP2*UVT(I))
2002  TWPCC=TWPCC+DP2(I)*CNNXM(TP3,I)
      TWPCC=EXP(-TWPCC)
C
C  INSURE THAT TWPCC IS LEGAL(>1.0E-16)
C
      IF(TWPCC.LT.1.0E-16)TWPCC=1.0E-16
C
C  CALCULATE CPT AND CQT
C
      DO 7100 I=1,NLYRS
      TP1=UV(I)*TOPRP-LOS(I)*LU/ZT
      TP2=VV(I)*TOPRP
7100  ARG(I)=SQRT(TP1*TP1*UT(I)+TP2*TP2*VT(I)-TP1*TP2*
     1 UVT(I))
      TP7=CNNXMI(NLYRS,0.1)
      TP7=1.0-CLIM/(2.0*TP7*TP7)
      DO 7200 I=1,NLYRS
      TP1=UV(I)*TOPRP
      TP2=VV(I)*TOPRP-LOS(I)*LV/ZT
7200  ARG(I)=SQRT(TP1*TP1*UT(I)+TP2*TP2*VT(I)-TP1*TP2*
     1 UVT(I))
      TP8=CNNXMI(NLYRS,0.1)
      TP8=1.0-CLIM/(2.0*TP8*TP8)
      CPT=LP*(TP7*TP21/LU+TP8*TP20/LV)
      CQT=LQ*(TP8*TP21/LV-TP7*TP20/LU)
C
C  INSURE THAT CPT AND CQT ARE LEGAL(<0.9998)
C
      X2=CPT*CPT+CQT*CQT
      IF(X2.GE.0.9998)THEN
          X2=SQRT(0.9998/X2)
          CPT=CPT*X2
          CQT=CQT*X2
      END IF
C
C  CALCULATE PARALLEL TO AND FINAL TO
C
```

```
      TOPAR=ABS((VTR(1)*LOSV(1)+VTR(2)*LOSV(2)+VTR(3)*LOSV(3))
     1  /LOSVA-VZ)/LPP**2+ABS((VRE(1)*LOSV(1)+VRE(2)*LOSV(2)
     1  +VRE(3)*LOSV(3))/LOSVA-VZ)/LP**2
      TOPAR=6.6E-5*F/(TOPAR+1.0E-20)
      IF(TOPAR.LT.TOPRP)THEN
          TO=TOPAR
      ELSE
          TO=TOPRP
      END IF
C
C  CALCULATE X2, THE MEAN SQUARE LOG AMPLITUDE FLUCTUATION
C
      X2=0.0
      XN(1)=-4.6
      XN(2)=-2.4
      XN(3)=-1.6
      XN(4)=-0.8
      XN(5)=0.0
      XN(6)=0.8
      XN(7)=1.6
      XN(8)=2.4
      DO 2150 I=1,NLYRS
      TP18=N(I)
      TP3=LO(I)*LO(I)
      TP1=TP3/LY2(I)
      TP2=(1.0-TP1)**2
      TP1=(1.0+TP1)**2
      TP3=SQRT(TP1+0.5*TP2)/(8.38E-05*F*TP3)
      TP2=TP2/TP1
      TP22=ALOG((1.0+0.5*TP2)**2/(1.2+4.8*TP2))
      TP23=(LF(I)/LO(I))**2
      TP24=NP(I)-TP18
      TP25=ALOG(LOS(I)*(ZT-LOS(I))*TP3/ZT)
      TP11=1.1-TP25
      TP10=0.5*TP22-TP25
      TP13=(0.69+TP22)/(TP10-TP11)
      L=15
      TP7=-ALOG(TP23)
      XN(9)=TP7-2.4
      XN(10)=TP7-1.6
```

```
      XN(11)=TP7-0.8
      XN(12)=TP7
      XN(13)=TP7+0.8
      XN(14)=TP7+1.6
      XN(15)=TP7+2.4
      TP6=-2.0*ALOG(LI(I)/LO(I))
      DO 2153 J=6,L
2153  IF(XN(J).GT.TP6)GO TO 2152
      J=L+1
2152  L=J
      XN(L)=TP6
      TP25=2.0*TP25
      TP5=3.0
      TP8=AMIN1(TP10,TP7)-4.6
      DO 2158 J=1,L
2158  IF(XN(J).GT.TP8)GO TO 2157
2157  TP7=TP10
      TP14=EXP(TP8)
      TP3=TP5*TP8+TP25-TP18*ALOG(1.0+TP14)-TP24*ALOG(1.0+TP23
     1 *TP14)
      TP14=EXP(TP3)
      IFLG=-1
      TP17=0.0
2160  TP1=XN(J)
      IF(TP1.LE.TP8)GO TO 2161
      IF(TP1.LT.TP7)GO TO 2162
      TP1=TP7
      J=J-1
      IF(IFLG)2171,2172,2172
2171  TP25=TP22-TP13*TP10
      TP5=TP13+1.0
      IFLG=1
      TP7=TP11
      GO TO 2162
2172  TP25=-0.69
      TP5=1.0
      TP7=1.0E+10
2162  TP16=EXP(TP1)
      TP15=TP25+TP5*TP1-TP18*ALOG(1.0+TP16)-TP24*ALOG(1.0+TP23
     1 *TP16)
```

```
      TP12=EXP(TP15)
      TP16=TP15-TP3
      IF(ABS(TP16).GT.0.0001)TP14=(TP12-TP14)/TP16
      TP17=TP17+(TP1-TP8)*TP14
      TP14=TP12
      TP8=TP1
      TP3=TP15
2161  J=J+1
      IF(J.LE.L)GO TO 2160
2150  X2=X2+Q2(I)*DP2(I)*(TP18-1.0)*TP17
C
C  CALCULATE FO
C
      TP1=(1.0/LU**4+1.0/LV**4+2.0*(CUV/(LU*LV))**2
     1 )**(-0.25)
      TP2=(1.0/LUP**4+1.0/LVP**4+2.0*(CUVP/(LUP*LVP))**2
     1 )**(-0.25)
      TP7=TP1*TP2
      DO 2100 I=1,NLYRS
      TP1=SQRT(TP7*LOS(I)*(ZT-LOS(I))*SQRT(UT(I)**2+VT(I)**2
     1 +0.5*UVT(I)**2))/ZT
      Q2(I)=DP2(I)*CNNXM(TP1,I)/(TP1*TP1*DLOS(I))
2100  ARG(I)=Q2(I)*UT(I)
      FO=FOINTR(NLYRS,ARG,LOS,DLOS)
      DO 2120 I=1,NLYRS
2120  ARG(I)=Q2(I)*VT(I)
      FO=FO+FOINTR(NLYRS,ARG,LOS,DLOS)
      DO 2140 I=1,NLYRS
2140  ARG(I)=0.5*Q2(I)*UVT(I)
      FO=FO+2.0*FOINTR(NLYRS,ARG,LOS,DLOS)
      IF(P2.GT.0.0)THEN
          FO=1.48E-05*F*F/SQRT(FO)
      ELSE
          FO=10.0*F
      END IF
      FO=FO/SQRT(1.0+C1*C1)
C
C  OUTPUT LP, LQ,,LPP, AND LQP IN METERS/SEC
C
      LP=TNPK*LP
```

```
      LQ=TMPK*LQ
      LPP=TMPK*LPP
      LQP=TMPK*LQP
      IF(MODE.EQ.0)RETURN
C
C  CALCULATE EFFECTIVE SCALES AND INDICES
C    USE DOPPLER WEIGHTING, TP12=2.0
C
      DP=TEC/(118.0*F)
      DL=DP/(6.3*F)
      DO 6741 I=1,4
      DLM(I)=0.0
6741  DPM(I)=0.0
      TP12=2.0
      TP5=0.0
      LOEF=0.0
      LFEF=0.0
      LIEF=0.0
      NEF=0.0
      NPEF=0.0
      TP6=0.0
      DO 2500 I=1,NLYRS
          TP4=SQRT(UV(I)*UV(I)+VV(I)*VV(I))
          IF(TP4 .GT. 0.0)THEN
              R=LF(I)/LO(I)
              RP=LI(I)/LF(I)
              QN=N(I)
              QNP=NP(I)
              TP3=SQRT(UV(I)*UV(I)*UT(I)+VV(I)*VV(I)*VT(I)
     1          -UV(I)*VV(I)*UVT(I))
              TP20=DP2(I)*TP3**TP12*GWEIGT(TP12)/GWEIGT(0.0)
              TP5=TP5+TP20
              TP6=TP6+TP20*LOS(I)*(ZT-LOS(I))*(TP3/TP4)**2
              ARG(I)=TP3*LOS(I)/(TP4*ZT)
              LOEF=LOEF+TP20/TP3
              LFEF=LFEF+TP20*R/TP3
              LIEF=LIEF+TP20*R*RP/TP3
              NEF=NEF+TP20*QN
              NPEF=NPEF+TP20*QNP
          ELSE
```

```
                ARG(I) = 0.0
            END IF
2500        CONTINUE
        IF(TP5.LE.0.0)THEN
            KR=0.0
            FR=0.0
            VEF=0.0
            RETURN
        END IF
        NEF=NEF/TP5
        NPEF=NPEF/TP5
        TP6=TP6/TP5
        VEF=CNNXMI(NLYRS,XLEST)/TOPRP
        TP7=VEF/TP5
        LOEF=LOEF*TP7
        LFEF=LFEF*TP7
        LIEF=LIEF*TP7
        LOEF=AMAX1(LFEF,LOEF)
        LIEF=AMIN1(LIEF,LOEF)
        R=LFEF/LOEF
        RP=LIEF/LFEF
        QN=NEF
        QNP=NPEF
        P2=TP5/((VEF/LOEF)**TP12*GWEIGT(TP12)/GWEIGT(0.0))
        IF(MODE.EQ.1)RETURN
C
C  CALCULATE RAYLEIGH WAVENUMBER AND FREQUENCY
C
        TP21=1.0+((0.06*NPEF-0.54)*NPEF+0.38)*ALOG(LFEF/LOEF)
        TP21=1.0/(1.0-(NPEF-NEF)*(TP21*LFEF/LOEF)**(NEF+NEF-2.0)
       1 /(NPEF-1.0))
        TP20=TP21*P2*(NEF-1.0)
C
C  APPROXIMATE ESTIMATES FOR TP25 AND VEF WHEN NOT AVAILABLE
C  DIRECTLY FROM THE EFFECTIVE SCALE, INDEX, AND VELOCITY
C  CALCULATION ARE
C     TP25=1.46E+04*ZT*LP/(LOEF**2*LPP*F)
C     VEF=LP/TO
C  THE STANDARD VALUE OF THE RAYLEIGH CRITERIA IS
C     RC=0.1
```

```
C  THE PHASE VARIANCE DEFAULT IS
C     PND=0.025
C  XN(I) FOR I FROM 1 TO 8 IS SET ABOVE AND DOES NOT NEED TO
C  BE RESET HERE
C
      TP25=1.46E+04*TP6/(F*ZT)
      TP10=-0.49-ALOG(TP25)
C     XN(1)=-4.6
      XN(1)=AMIN1(TP10,XN(1))-2.3
      TP17=TP20*TP25**2*EXP(3.0*XN(1))/3.0
      IF(TP17 GT.RC)THEN
         KR=(3.0*RC/(TP20*TP25**2))**(1.0/3.0)
      ELSE
         TP25=ALOG(TP25)
         TP18=NEF
         TP23=(LFEF/LOEF)**2
         TP24=NPEF-NEF
C        XN(2)=-2.4
C        XN(3)=-1.6
C        XN(4)=-0.8
C        XN(5)=0.0
C        XN(6)=0.8
C        XN(7)=1.6
C        XN(8)=2.4
         L=15
         TP7=-ALOG(TP23)
         XN(9)=TP7-2.4
         XN(10)=TP7-1.6
         XN(11)=TP7-0.8
         XN(12)=TP7
         XN(13)=TP7+0.8
         XN(14)=TP7+1.6
         XN(15)=TP7+2.4
         TP6=-2.0*ALOG(LIEF/LOEF)
         DO 4153 J=6,L
4153        IF(XN(J).GT.TP6)GO TO 4152
         J=L+1
4152     L=J
         XN(L)=TP6
         TP11=1.1-TP25
```

```
      TP13=-0.291/(TP10-TP11)
      TP25=2.0*TP25
      TP5=3.0
      TP8=XN(1)
      TP7=TP10
      TP14=EXP(TP8)
      TP3=TP5*TP8+TP25-TP18*ALOG(1.0+TP14)-TP24*ALOG(1.0+TP23
     1  *TP14)
      TP14=EXP(TP3)
      IFLG=-1
      J=2
4160  TP1=XN(J)
      IF(TP1.LE.TP8)GO TO 4161
      IF(TP1.LT.TP7)GO TO 4162
      TP1=TP7
      J=J-1
      IF(IFLG)4176,4172,4172
4176  TP25=-0.9808-TP13*TP10
      TP5=TP13+1.0
      IFLG=1
      TP7=TP11
      GO TO 4162
4172  TP25=-0.69
      TP5=1.0
      TP7=1.0E+10
4162  TP16=EXP(TP1)
      TP15=TP25+TP5*TP1-TP18*ALOG(1.0+TP16)-TP24*ALOG(1.0+TP23
     1  *TP16)
      TP16=TP15-TP3
      TP12=EXP(TP15)
      IF(ABS(TP16).GT.0.0001)TP14=(TP12-TP14)/TP16
      TP19=TP20*TP14*(TP1-TP8)
      IF((TP17+TP19).GE.RC)GO TO 4173
      TP17=TP17+TP19
      TP14=TP12
      TP8=TP1
      TP3=TP15
4161  J=J+1
      IF(J.LE.L)GO TO 4160
      KR=1.0E+30
```

```
          GO TO 4171
4173      TP16=TP16/(TP1-TP8)
          KR=(RC-TP17)/(TP20*EXP(TP3))
          IF(ABS(TP16*KR).GT.0.0001)THEN
             KR=EXP(TP8)*(1.0+TP16*KR)**(1.0/TP16)
          ELSE
             KR=EXP(TP8+KR)
          END IF
       END IF


4171  KR=SQRT(KR)/LOEF
C
C  KR=RAYLEIGH WAVE NUMBER FROM FLUCTUATIONS
C
C  FIGURE OUT PHASE NOISE LIMITED RAYLEIGH WAVE NUMBER
C
      TP1=(LFEF/LOEF)**1.2
      IF(NEF.NE.NPEF)GO TO 5105
      TP2=EXP(-6.4/(NEF*(NEF+6.4)))
      GO TO 5110
5105  TP2=((1.0+6.4/NPEF)/(1.0+6.4/NEF))**(1.0/(NPEF-NEF))
5110  TP1=TP1+(1.0-TP1)*TP2
      TP9=LFEF/SQRT(TP1)
      TP21=0.24*(NEF-1.0)*TP21*(1.0+6.4/NPEF)*P2*LOEF/TP1**
     1 (NPEF-NEF)
      TP2=TP21
      TP3=1.0/LOEF
      TP4=1.0/TP9
      TP2=TP2/(TP9**(2.0*(NPEF-NEF))*LOEF**(2.0*NEF-1.0))
      TP5=2.0-NPEF-NPEF
      TP6=(-PND*TP5/TP2)**(1.0/TP5)
      IF(TP6.GT.TP4)GO TO 5227
      TP7=-TP2*TP4**TP5/TP5
      TP2=TP2*TP9**(2.0*(NPEF-NEF))
      TP5=2.0-NEF-NEF
      TP6=(TP4**TP5+(TP7-PND)*TP5/TP2)**(1.0/TP5)
      IF(TP6.GT.TP3)GO TO 5227
      TP7=TP7+TP2*(TP4**TP5-TP3**TP5)/TP5
      TP2=TP2*LOEF**(NEF+NEF-1.0)
```

```
      TP6=TP3+(TP7-PND)/TP2
      IF(TP6.LT.0.0)TP6=0.0
5227  IF(TP6.LT.KR)KR=TP6
      FR=VEF*KR/6.3
C
C  KR=FINAL RAYLEIGH WAVE NUMBER
C  FR=RAYLEIGH FREQUENCY
C
C
C  DO DYNAMICS QUANTITIES, THAT IS, CALCULATE THE THREE SIGMA
C  VALUES FOR DOPPLER, DOPPLER RATE, GROUP DELAY, ETC.
C
      IF(KR.LE.0.0)GO TO 6742
      TP25=NEF-0.5
      TP22=LOEF*LOEF
      TP24=NPEF-NEF
      TP23=LFEF*LFEF/TP1
      L=15
      TP3=-ALOG(LOEF)
      XN(5)=TP3
      TP6=0.5*ALOG(TP1)-ALOG(LFEF)
      XN(12)=TP6
      DO 6770 J=1,3
      TP7=0.4*FLOAT(J)
      XN(J+5)=TP3+TP7
      XN(J+12)=TP6+TP7
      N=5-J
      XN(N)=TP3-TP7
6770  XN(N+7)=TP6-TP7
      TP8=-ALOG(LIEF)
      DO 6707 I=6,L
6707  IF(XN(I).GT.TP8)GO TO 6713
      I=L+1
6713  L=I
      XN(L)=TP8
      TP3=ALOG(KR)
      XN(1)=AMIN1(XN(2),TP3)-4.6
      DO 6260 N=1,4
      I=2*(N-1)+1
      TP7=XN(I)
```

```
      TP6=EXP(2.0*TP7)
      TP6=TP7*FLOAT(I)-TP25*ALOG(1.0+TP22*TP6)-TP24*ALOG(
     1 1.0+TP23*TP6)
      TP17=EXP(TP6)
      J=2
6265  TP1=XN(J)
      IF(TP1.LE.TP7)GO TO 6271
      IF(TP1.GE.TP3)TP1=TP3
      TP15=EXP(2.0*TP1)
      TP15=TP1*FLOAT(I)-TP25*ALOG(1.0+TP22*TP15)-TP24*ALOG(
     1 1.0+TP23*TP15)
      TP16=EXP(TP15)
      TP12=TP15-TP6
      IF(ABS(TP12).GT.0.0001)TP17=(TP16-TP17)/TP12
      DPM(M)=DPM(M)+TP17*(TP1-TP7)
      IF(ABS((TP1-TP3)/TP3).LT.1.0E-3)GO TO 6260
      TP17=TP16
      TP7=TP1
      TP6=TP15
6271  J=J+1
      IF(J.LE.L)GO TO 6265
6260  DPM(M)=TP21*DPM(M)
      DO 6305 M=1,4
      J=M-1
6305  DPM(M)=VEF**J*SQRT(DPM(M))/2.1
6742  DPM(1)=6.3*DPM(1)
      DLM(1)=DPM(1)/(6.3*F)
      DO 6330 J=2,4
6330  DLM(J)=DPM(J)/F
      IF(MODE.EQ.2)RETURN
      END
      FUNCTION GWEIGT(M)
C
C  THIS FUNCTION IS A WEIGHTING FUNCTION FOR CALCULATING
C  EFFECTIVE SCALES AND INDICES.
C
      REAL M
      COMMON /CMDATA/QN,QNP,R,RP
      TP3=M+2.0-2.0*QN
      IF(ABS(TP3).LT.1.0E-4)GO TO 10
```

```
      TP1=(1.0-R**TP3)/TP3
      GO TO 20
10    TP1=-ALOG(R)
20    TP2=2.0*QNP-M-2.0
      IF(ABS(TP2).LT.1.0E-4)GO TO 30
      TP2=(1.0-RP**TP2)/TP2
      GO TO 40
30    TP2=-ALOG(RP)
40    GWEIGT=1.0/(M+1.0)+(TP1+TP2)/R**TP3
      RETURN
      END
      FUNCTION FOINTR(NLYRS,ARG,Z,DZ)
C
C  THIS FUNCTION INTEGRATES THE LOS INTEGRALS FOR SOLVING FOR FO.
C  THIS IS A TRICKY INTEGRAL. DO NOT FOOL WITH THIS ROUTINE.
C
      DIMENSION ARG(1),Z(1),DZ(1)
      FOINTR=0.0
      A8=0.0
      ZT=Z(NLYRS+1)
      DO 1 I=1,NLYRS
      A5=(Z(I)-0.5*DZ(I))/ZT
      A6=1.0-A5
      A7=DZ(I)/ZT
      FP=ARG(I)
      FOINTR=FOINTR+FP*(((FP*(((A7/18.0+0.2*A5-A6/7.5)*A7+A6*A6
     1 /12.0+0.25*A5*A5-0.5*A5*A6)*A7+A6*A6*A5/3.0-2.0*A6*A5*A5
     2 /3.0)+A8/3.0)*A7+0.5*FP*(A5*A6)**2-A6*A8)*A7+A8*A6*A6)*A7
1     A8=A8+FP*((A7/3.0+A5)*A7+A5*A5)*A7
      FOINTR=2.0*FOINTR*ZT**4
      RETURN
      END
      FUNCTION CNNXMI(NLYRS,TPX)
C
C  THIS IS A CRITICAL FUNCTION.  IT ITERATES OVER A LOS
C  INTEGRATION TO FIND DECORRELATION DISTANCES, TIMES, ETC.
C  DO NOT, I REPEAT, DO NOT MAKE CRIT GREATER THAN 0.001.
C  THIS WOULD COMPROMISE THE ACCURACY SPECIFICATION ON THE
C  ITERATION.  SMALLER VALUES DO NOT BUY MUCH EITHER.
C
```

```
      PARAMETER (CRIT=0.001,EM1=0.36787944,EM2=0.63212055)
      PARAMETER (NIP=11)
      DIMENSION ARG(NIP),DP2(NIP)
      COMMON /CNNCOM/CLIM,ARG,DP2
      TP3=0.0
      TP4=0.0
      TP1=TPX
      DO 1 I=1,NLYRS
      IF(ARG(I).GT.0.0)THEN
          TP4=TP4+DP2(I)
          TP5=TP1*ARG(I)
          TP3=TP3+DP2(I)*CNNXM(TP5,I)
      END IF
1     CONTINUE
      IF(TP4.GT.1.0E-4)GO TO 10
      IF(TP4.LE.0.0)GO TO 50
      CLIN=EM2*TP4
      GO TO 20
10    CLIN=-ALOG(EM1+EM2*EXP(-TP4))
20    CNNXMI=SQRT(TP3/CLIN)/TP1
      IF(CNNXMI.LT.1.0E-08)GO TO 50
      CNNXMI=1.0/CNNXMI
4     TP4=0.0
      DO 2 I=1,NLYRS
      TP5=CNNXMI*ARG(I)
2     TP4=TP4+DP2(I)*CNNXM(TP5,I)
      TP4=TP4/CLIN
      IF(ABS(TP4-1.0).LT.CRIT)RETURN
      TP5=ALOG(CNNXMI/TP1)/ALOG(TP4/TP3)
      IF(TP5 .GT. 1.6)TP5=1.6
      TP1=CNNXMI
      TP3=TP4
      CNNXMI=CNNXMI/TP4**TP5
      GO TO 4
50    CNNXMI=1.0E+8
      RETURN
      END
      FUNCTION CNNXM(X,IFLG)
C
C FUNCTION CNNXM, VERSION 3.0, 4 APR 89
```

```
C
C THIS IS THE PHASE STRUCTURE FUNCTION FOR THE WITTWER-KILB POWER
C SPECTRUM.  WHEN USED TO CALCULATE THE SIGNAL DECORRELATION
C DISTANCE, THE MAXIMUM ERROR IN THE DISTANCE IS FIVE PERCENT.
C THE MAXIMUM ERROR IN THE STRUCTURE FUNCTION, ITSELF, IS ABOUT TEN
C PERCENT.
C
C THE INPUT VARIABLES ARE:
C
C   LABELED COMMON BLOCK /CNDATA/
C    (THESE VARIABLES ARE USED ONLY WHEN IFLG < 0)
C      2*N-2 = INTERMEDIATE SCALE SPECTRAL INDEX
C              (N.GE.1.6.AND.N.LE.2.0)
C      2*NP-2 = SMALL SCALE SPECTRAL INDEX
C               (NP.GE.2.0.AND.NP.LE.4.0)
C      R = RATIO OF FREEZING TO OUTER SCALE
C          (R.LE.1.0.AND.R.GE.1.0E-07)
C      RP = RATIO OF INNER TO OUTER SCALE
C           (RP.LE.R)
C
C   FORMAL ARGUMENTS
C      X = DISPLACEMENT DIVIDED BY OUTER SCALE
C      IABS(IFLG) = INDEX OF LOS INTEGRATION POINT
C                   (IABS(IFLG).LE.NIP.AND.IFLG.NE.0)
C      IFLG < 0, CALCULATE AND STORE INTERMEDIATE RESULTS.  THIS
C                IS USED WHENEVER N, NP, R, OR RP CHANGES FOR
C                SOME VALUE OF IABS(IFLG).
C      IFLG > 0, USE STORED INTERMEDIATE DATA.  ASSUMES THAT
C                N, NP, R, AND RP HAVE NOT CHANGED FOR CURRENT
C                VALUE OF IABS(IFLG) SINCE LAST CALL.
C
      SAVE
      REAL N,NP,L2
C
C   NIP=NUMBER OF LOS INTEGRATION POINTS
C
      PARAMETER (NIP=11,E2=-7.1509297E-01)
      PARAMETER (D3=6.1788469E-01,C3=9.5310179E-02)
      PARAMETER (D4=1.3862943,C1=-6.9314718E-01)
      PARAMETER (E7=2.8490703E-01,C4=6.5551593E-01)
```

```
      DIMENSION XPSD(12*NIP),FP(NIP),E4(NIP),JP(NIP)
      DIMENSION E5(NIP),L2(NIP),IA7(NIP),JM(NIP)
      DIMENSION XPSDP(12*NIP),XPSDM(12*NIP),XN(12*NIP)
      COMMON /CNDATA/N,NP,R,RP
      I=IABS(IFLG)
      JSTART=(I-1)*12+1
      IF(IFLG.LT.0)THEN
C
C DO A3=d0**2
C
         A3=R**1.2
         IF(ABS(N-NP).LT.0.001)THEN
            A6=EXP(-6.4/(N*(N+6.4)))
         ELSE
            A6=((1.0+6.4/NP)/(1.0+6.4/N))**(1.0/(NP-N))
         END IF
         A3=A3+(1.0-A3)*A6
C
C INITIAL NORMALIZATION CONSTANT
C
         A6=1.00+((0.06*NP-0.54)*NP+0.38)*ALOG(R)
         A6=(A6*R)**(2.0*N-2.0)
         A6=1.0/(1.0-(NP-N)*A6/(NP-1.0))
         FP(I)=0.24*A6*(N-1.0)*(1.0+6.4/NP)/A3**(NP-N)
C
C CONSTANTS
C
         L2(I)=R*R/A3
C        XO=ALOG(1.0/SQRT(L2))
         XO=-0.5*ALOG(L2(I))
         IF(XO.LT.2.05)THEN
            A3=0.951220*XO-1.0
            XN(JSTART+1)=A3-2.3
            XN(JSTART+2)=A3-1.5
            XN(JSTART+3)=A3-1.0
            XN(JSTART+4)=A3-0.5
            XN(JSTART+5)=A3
            XN(JSTART+6)=A3+0.5
            XN(JSTART+7)=A3+1.0
            XN(JSTART+8)=A3+1.55
```

```
          JEND=JSTART+9
          XN(JEND)=A3+2.45
      ELSE
          XN(JSTART+1)=-1.35
          XN(JSTART+2)=-0.55
          XN(JSTART+3)=-0.05
          XN(JSTART+4)=0.45
          J=JSTART+5
          IF(XO.GT.2.85)THEN
              XN(J)=1.35
              XN(J+1)=XO-1.49999
              XN(J+2)=XO-0.60
              JEND=J+5
          ELSE
              XN(J+1)=XO-0.60
              XN(J)=0.5*(XN(J+1)+XN(J-1))
              JEND=J+4
          END IF
          XN(JEND-2)=XO-0.10
          XN(JEND-1)=XO+0.45
          XN(JEND)=XO+1.35
      END IF
      A4=-ALOG(RP)
807   IF(XN(JEND).LT.A4)GO TO 812
      JEND=JEND-1
      GO TO 807
812   JEND=JEND+1
      XN(JEND)=A4
      IA7(I)=JEND
      XPSDM(JEND)=0.0
      JM(I)=JEND
      E4(I)=N-0.5
      E5(I)=NP-N
      XPSDP(JSTART)=0.0
      JP(I)=JSTART
      XPSD(JSTART)=0.0
      JS=JSTART+1
      DO 900 J=JS,JEND
          A3=EXP(2.0*XN(J))
900       XPSD(J)=E4(I)*ALOG(1.0+A3)+E5(I)*ALOG(1.0+
```

```
     1           L2(I)*A3)-XN(J)
C
C  2*SIN(K*X/2)**2 = C1*(K*X)**2   , K<=DO
C                  = C2*K**E2      , DO<K<=D1
C                  = C3            , D1<K
C
C  C1 = COEFFICIENT FOR SMALL (K*X)**2 = 0.5
C  D3 = PEAK VALUE OF APPROXIMATE FUNCTION = 1.855
C  DO = K AT APPROXIMATE FUNCTION PEAK = SQRT(D3/(C1*X**2))
C  D1 = K AT END OF OVERSHOOT = D4/X, D4 = 4.0
C  C2 = COEFFICIENT FOR INTERMEDIATE K = D3/DO**E2
C  C3 = APPROXIMATE FUNCTION AT LARGE K = 1.10
C  E2 = EXPONENT FOR INTERMEDIATE K = ALOG(C3/D3)/ALOG(D1/DO)
C
C         D3=ALOG(1.855)
C         C3=ALOG(1.10)
C         D4=ALOG(4.0)
C         E2=(C3-D3)/(D1-DO)
C         E2=(C3-D3)/(D4-0.5*(D3-C1))
C         C1=ALOG(0.5)
C         C4=0.5*(D3-C1)
C         E7=E2+1.0
C
      END IF
      CNNXN=0.0
      IF(X.EQ.0.0)RETURN
      XX=ALOG(ABS(X))
      X2=X*X
C     DO=0.5*(D3-XX-XX-C1)
C     D1=D4-XX
C     E2=(C3-D3)/(D1-DO)
C     E7=1.0+E2
      DO=C4-XX
      XN(JSTART)=AMIN1(DO,XN(JSTART+1))-5.6
      XPSD(JSTART)=-XN(JSTART)
      J=IA7(I)
      IF(XN(J).GT.DO)THEN
         D1=D4-XX
         C2=D3-E2*DO
         IF(XN(J).GT.D1)THEN
```

```
          JE=J-1
          JT=JE+JSTART
          DO 800 JJ=JSTART,JE
              J=JT-JJ
800           IF(XN(J).LT.D1)GO TO 850
850       J=J+1
          IF(J.LT.JM(I))THEN
              JE=JM(I)-1
              JT=JE+J
              A4=C3-XPSD(JE+1)
              FO=EXP(A4)
              DO 870 JJ=J,JE
                  JS=JT-JJ
                  FN=FO
                  A6=A4
                  A4=C3-XPSD(JS)
                  FO=EXP(A4)
                  AO=A6-A4
                  IF(ABS(AO).GT.0.0001)FN=(FN-FO)/AO
870               XPSDM(JS)=XPSDM(JS+1)+FN*(XN(JS+1)-XN(JS))
              JM(I)=J
          END IF
          CNNXM=XPSDM(J)
          CT=EXP(D1+D1)
          CT=E4(I)*ALOG(1.0+CT)+E5(I)*ALOG(1.0+L2(I)*CT)
          A6=C3+D1-CT
          FO=EXP(A6)
          A4=C3-XPSD(J)
          A1=EXP(A4)
          AO=A4-A6
          IF(ABS(AO).GT.0.0001)A1=(A1-FO)/AO
          CNNXM=CNNXM+A1*(XN(J)-D1)
      ELSE
          D1=XN(J)
          CT=EXP(D1+D1)
          CT=E4(I)*ALOG(1.0+CT)+E5(I)*ALOG(1.0+L2(I)*CT)
          A6=C2+E7*D1-CT
          FO=EXP(A6)
      END IF
875   J=J-1
```

```
          IF(XN(J).LT.DO)GO TO 876
          FN=FO
          A4=C2+E2*XN(J)-XPSD(J)
          FO=EXP(A4)
          AO=A6-A4
          IF(ABS(AO).GT.0.0001)FN=(FN-FO)/AO
          CNNXM=CNNXM+FN*(D1-XN(J))
          D1=XN(J)
          A6=A4
          GO TO 875
876       CT=EXP(DO+DO)
          CT=E4(I)*ALOG(1.0+CT)+E5(I)*ALOG(1.0+L2(I)*CT)
          A4=C2+E7*DO-CT
          AO=A6-A4
          IF(ABS(AO).GT.0.0001)FO=(FO-EXP(A4))/AO
          CNNXM=CNNXM+FO*(D1-DO)
          A6=C1+3.0*DO-CT
          A4=C1+2.0*XN(J)-XPSD(J)
          A1=EXP(A6)
          AO=A6-A4
          IF(ABS(AO).GT.0.0001)A1=(A1-EXP(A4))/AO
          CNNXM=CNNXM+X2*A1*(DO-XN(J))
      END IF
      IF(J.GT.JP(I))THEN
          JS=JP(I)
          A6=C1+2.0*XN(JS)-XPSD(JS)
          FN=EXP(A6)
          JS=JS+1
          DO 970 JJ=JS,J
              A4=A6
              FO=FN
              A6=C1+2.0*XN(JJ)-XPSD(JJ)
              FN=EXP(A6)
              AO=A6-A4
              IF(ABS(AO).GT.0.0001)FO=(FN-FO)/AO
970           XPSDP(JJ)=XPSDP(JJ-1)+FO*(XN(JJ)-XN(JJ-1))
          JP(I)=J
      END IF
      CNNXM=CNNXM+X2*XPSDP(J)
      CNNXM=FP(I)*CNNXM
```

```
      RETURN
      END
```

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